Shape Analysis & Measurement

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Shape Analysis & Measurement

- The extraction of quantitative feature information from images is the objective of image analysis.
- The objective may be:
  - shape quantification
  - count the number of structures
  - characterize the shape of structures

Shape Measures

- The most common object measurements made are those that describe shape.
  - Shape measurements are physical dimensional measures that characterize the appearance of an object.
  - The goal is to use the fewest necessary measures to characterize an object adequately so that it may be unambiguously classified.

Shape Measures

- The performance of any shape measurements depends on the quality of the original image and how well objects are pre-processed.
  - Object degradations such as small gaps, spurs, and noise can lead to poor measurement results, and ultimately to misclassifications.
  - Shape information is what remains once location, orientation, and size features of an object have been extracted.
  - The term pose is often used to refer to location, orientation, and size.

Shape Descriptors

- What are shape descriptors?
  - Shape descriptors describe specific characteristics regarding the geometry of a particular feature.
  - In general, shape descriptors or shape features are some set of numbers that are produced to describe a given shape.

Shape Descriptors

- The shape may not be entirely reconstructable from the descriptors, but the descriptors for different shapes should be different enough that the shapes can be discriminated.
- Shape features can be grouped into two classes: *boundary features* and *region features*.

Distances

- The simplest of all distance measurements is that between two specified pixels (x_1,y_1) and (x_2,y_2).
- There are several ways in which distances can be defined:
  - *Euclidean*

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

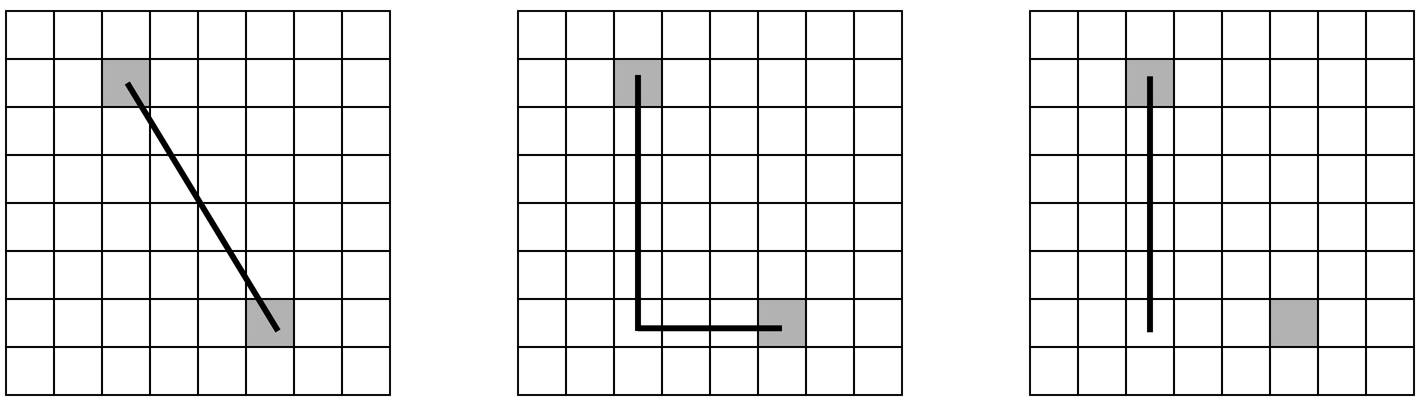
  - *Chessboard*

$$d = \max\left(\left|x_1 - x_2\right|, \left|y_1 - y_2\right|\right)$$

Distances

– *City-block*

$$d = |x_1 - x_2| + |y_1 - y_2|$$



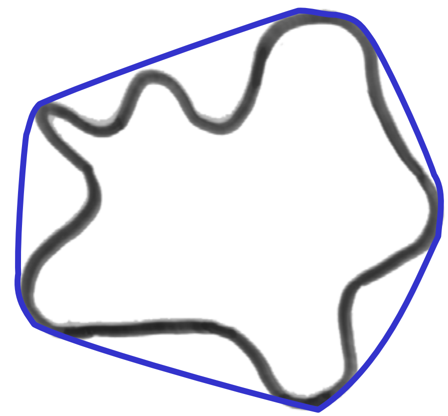
Euclidean
Chessboard

City-block

Area

- The area is the number of pixels in a shape.

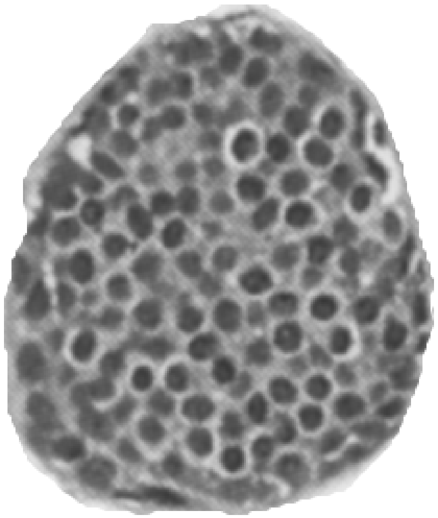
Net Area



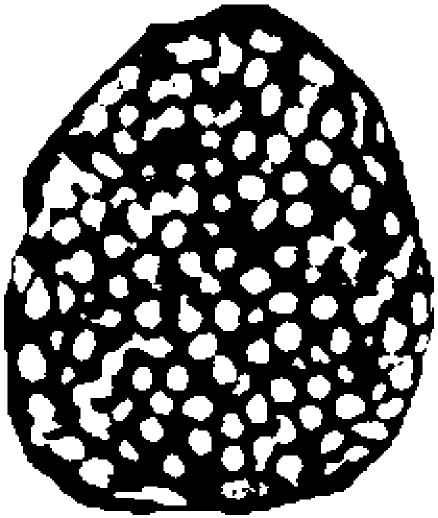
Convex Area

- The convex area of an object is the area of the convex hull that encloses the object.

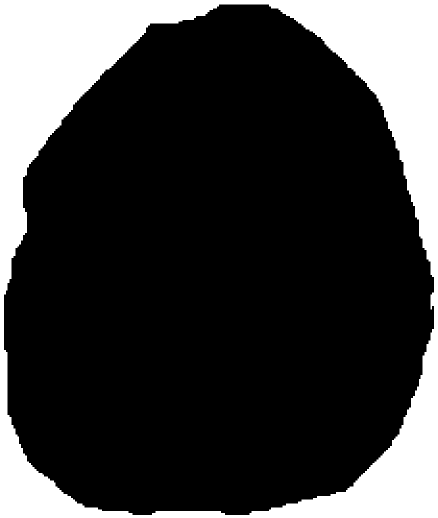
Area



Original Image



Net Area



Filled Area

Perimeter

- The perimeter [length] is the number of pixels in the boundary of the object.
  - If $x_1,\ldots,x_N$ is a boundary list, the perimeter is given by:

$$\text{perimeter} = \sum_{i=1}^{N-1} d_i = \sum_{i=1}^{N-1} \left| x_i - x_{i+1} \right|$$

  - The distances d_i are equal to 1 for 4-connected boundaries and 1 or $\sqrt{2}$ for 8-connected boundaries.

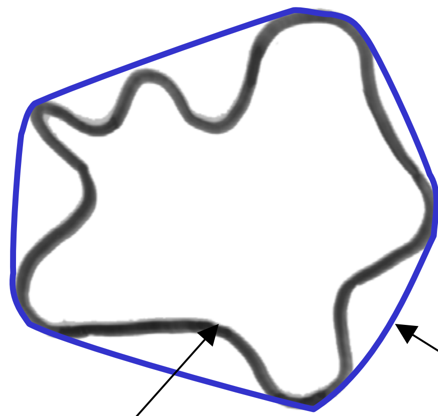
Perimeter

- For instance in an 8-connected boundary, the distance between diagonally adjacent pixels is the Euclidean measure $\sqrt{2}$
  - The number of diagonal links in $N_4–N_8$, and the remaining $N_8–(N_4–N_8)$ links in the 8-connected boundary are of one pixel unit in length. Therefore the total perimeter is:

$$\text{perimeter} = (\sqrt{2} - 1)N_4 + (2 - \sqrt{2})N_8$$

Perimeter

- The convex perimeter of an object is the perimeter of the convex hull that encloses the object.



Major Axis

- The major axis is the (x,y) endpoints of the longest line that can be drawn through the object.
  - The major axis endpoints (x_1,y_1) and (x_2,y_2) are found by computing the pixel distance between every combination of border pixels in the object boundary and finding the pair with the maximum length.

Major Axis Length

- The major-axis length of an object is the pixel distance between the major-axis endpoints and is given by the relation:

$$\text{major-axis length} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

  – The result is measure of object length.

Major Axis Angle

- The major-axis angle is the angle between the major-axis and the x-axis of the image:

$$\text{major-axis angle} = \tan^{-1}\left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

  – The angle can range from 0° to 360°.
  – The result is a measure of object orientation.

Minor Axis

- The minor axis is the (x,y) endpoints of the longest line that can be drawn through the object whilst remaining perpendicular with the major-axis.
  - The minor axis endpoints (x_1,y_1) and (x_2,y_2) are found by computing the pixel distance between the two border pixel endpoints.

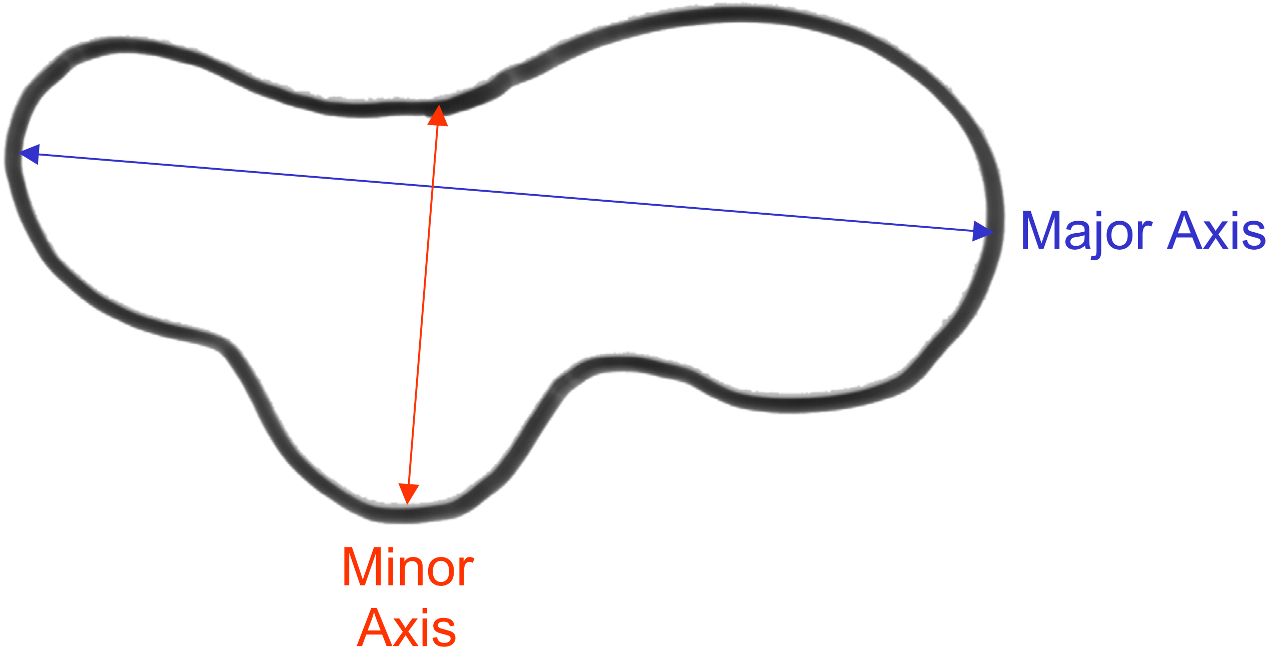
Minor Axis Length

- The minor-axis length of an object is the pixel distance between the minor-axis endpoints and is given by the relation:

$$\text{minor-axis length} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

  – The result is measure of object width.

Major and Minor Axes



Compactness

- Compactness is defined as the ratio of the area of an object to the area of a circle with the same perimeter.

$$\text{compactness} = \frac{4\pi \cdot area}{(perimeter)^2}$$

  - A circle is used as it is the object with the most compact shape.
  - The measure takes a maximum value of 1 for a circle
  - A square has compactness = $\pi/4$

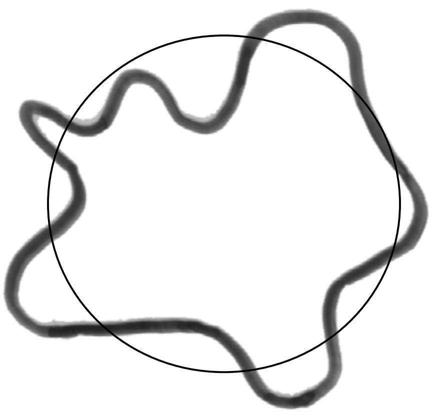
Compactness

- Objects which have an elliptical shape, or a boundary that is irregular rather than smooth, will decrease the measure.
- An alternate formulation:

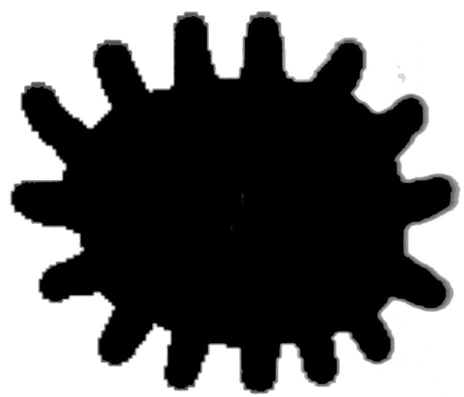
$$\text{compactness} = \frac{(perimeter)^2}{4\pi \cdot area}$$

- The measure takes a minimum value of 1 for a circle
- Objects that have complicated, irregular boundaries have larger compactness.

Compactness



low compactness



compactness=0.764

compactness=0.668

Elongation

- In its simplest form elongation is the ratio between the length and width of the object bounding box:

$$\text{elongation} = \frac{\text{width}_{\text{bounding-box}}}{\text{length}_{\text{bounding-box}}}$$

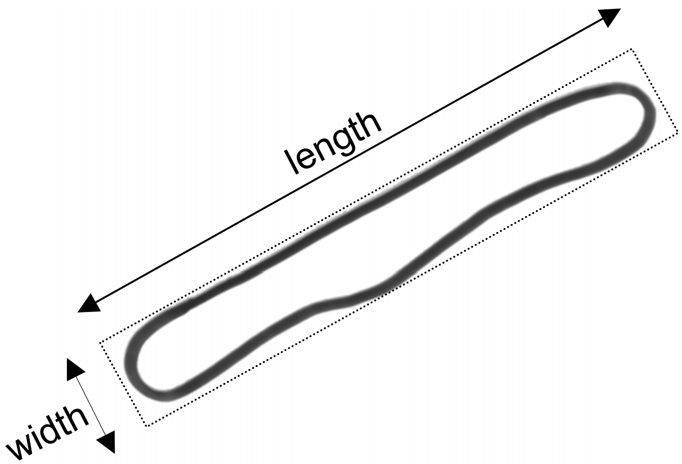
  - The result is a measure of object elongation, given as a value between 0 and 1.
  - If the ratio is equal to 1, the object is roughly square or circularly shaped. As the ratio decreases from 1, the object becomes more elongated.

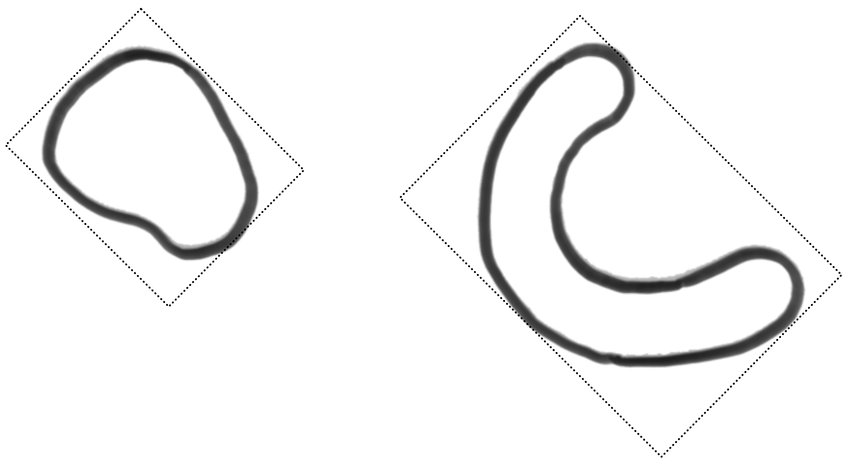
Elongation

- This criterion cannot succeed in curved regions, for which the evaluation of elongatedness must be based on maximum region thickness.
  - Elongatedness can be evaluated as a ratio of the region area and the square of its thickness.
  - The maximum region thickness (holes must be filled if present) can be determined as the number **d** of erosion steps that may be applied before the region totally disappears.

$$\text{elongation} = \frac{\textit{area}}{2\mathbf{d}^2}$$

Elongation





high elongation

low elongation

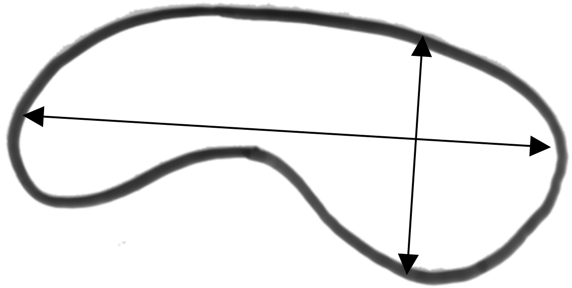
Eccentricity

- Eccentricity is the ratio of the length of the short (minor) axis to the length of the long (major) axis of an object:

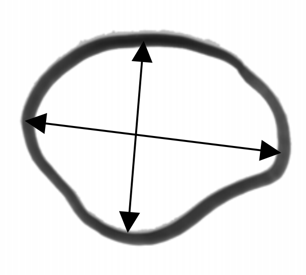
$$\text{eccentricity} = \frac{\text{axislength}_{\text{short}}}{\text{axislength}_{\text{long}}}$$

  - The result is a measure of object eccentricity, given as a value between 0 and 1.
  - Sometimes known as ellipticity.

Eccentricity



high eccentricity



low eccentricity

Eccentricity

- Eccentricity can also be calculated using central moments:

$$\text{eccentricity} = \frac{(\mu_{02} - \mu_{20})^2 + 4\mu_{11}}{\text{area}}$$

Measures of “Circularity”

- Sometimes it is useful to have measures that are sensitive only to departures of a certain type of circularity:

  e.g. convexity (measures irregularities)

  roundness (excludes local irregularities)

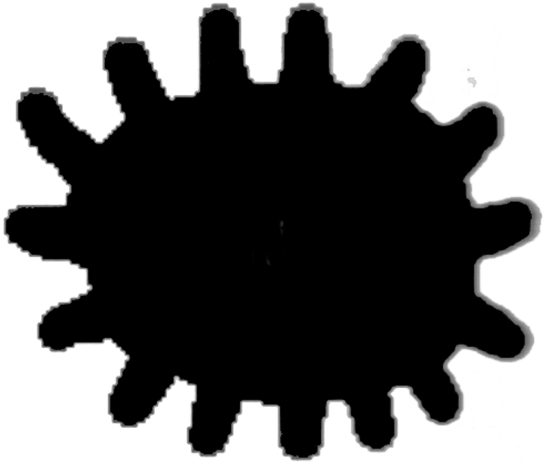
Circularity or Roundness

- A measure of roundness or circularity (area-to-perimeter ratio) which excludes local irregularities can be obtained as the ratio of the area of an object to the area of a circle with the same convex perimeter:

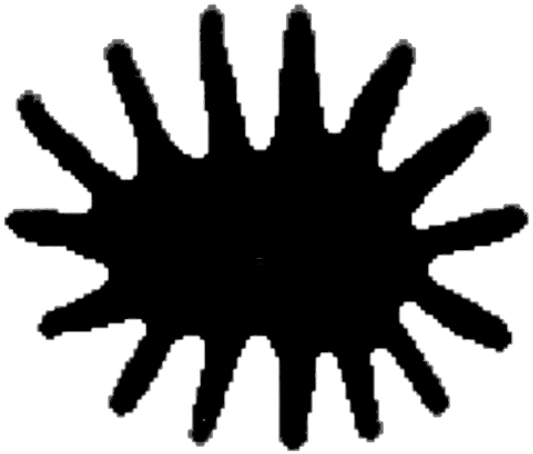
$$\text{roundness} = \frac{4\pi \cdot area}{(convex\ perimeter)^2}$$

  - This statistic equals 1 for a circular object and less than 1 for an object that departs from circularity, except that it is relatively insensitive to irregular boundaries.

Circularity



roundness=0.584

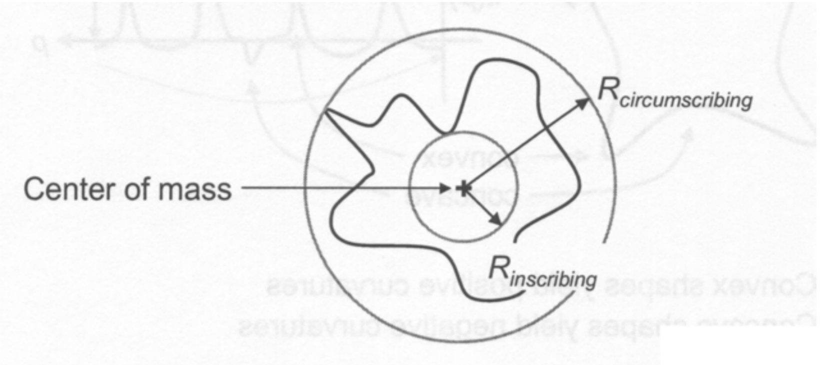


roundness=0.447

Sphericity

- Sphericity measures the degree to which an object approaches the shape of a “sphere”.

$$\text{sphericity} = \frac{R_{inscribing}}{R_{circumscribing}}$$



  – For a circle, the value is the maximum of 1.0

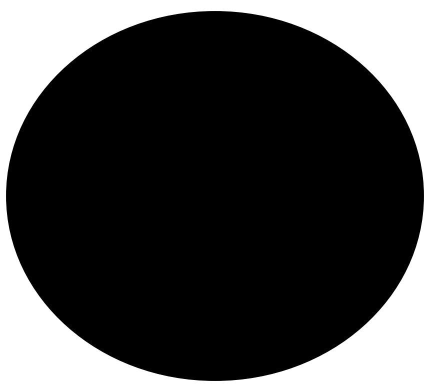
Convexity

- Convexity is the relative amount that an object differs from a convex object.
  - A measure of convexity can be obtained by forming the ratio of the perimeter of an object's convex hull to the perimeter of the object itself:

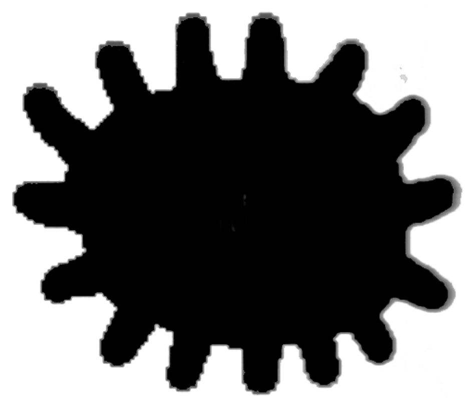
$$\text{convexity} = \frac{\text{convex perimeter}}{\text{perimeter}}$$

Convexity

– This will take the value of 1 for a convex object, and will be less than 1 if the object is not convex, such as one having an irregular boundary.



convexity=1.0



convexity=0.483

convexity=0.349

Aspect Ratio

- The aspect ratio measures the ratio of the objects height to its width.

$$\text{aspect ratio} = \frac{\text{height}}{\text{width}}$$

Caliper Dimensions

- Caliper or feret diameters are the distances between parallel tangents touching oppoisite sides of an object.
  - At orientation θ, the caliper diameter is:

$$\max_{(x,y)\in A}(x\sin\theta + y\cos\theta) - \min_{(xy)\in A}(x\sin\theta + y\cos\theta)$$

  - Certain caliper diameters are of special interest:
    - The width of an object ($\theta = 0°$)

$$\max_{(x,y)\in A}(y) - \min_{(x,y)\in A}(y)$$

Caliper Dimensions

- The height of an object ($\theta = 90°$)

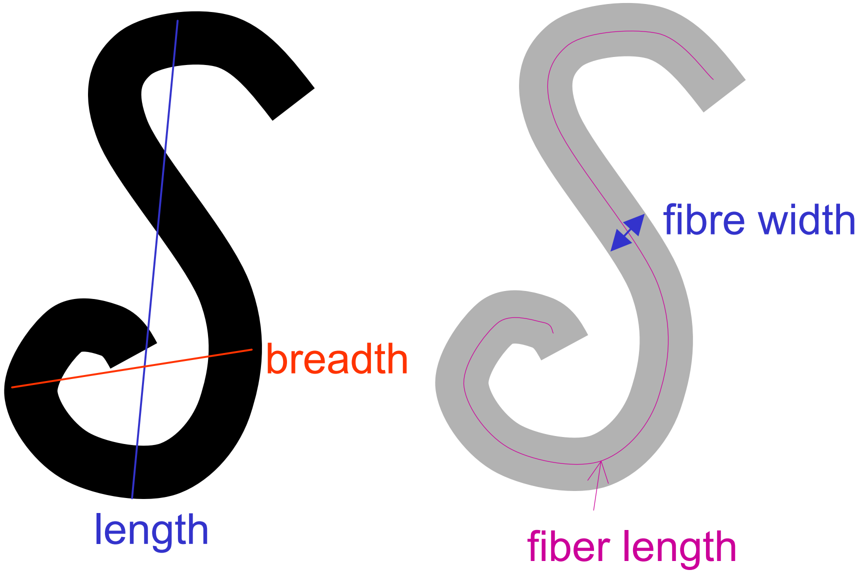
$$\max_{(x,y)\in A}(x) - \min_{(x,y)\in A}(x)$$

- The maximum caliper diameter is one definition of an objects length.

Curl

- The curl of an object measures the degree to which an object is “curled up”.

$$\text{curl} = \frac{\text{length}}{\text{fibre length}}$$



Curl

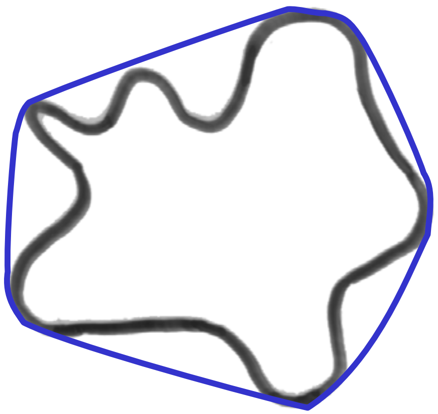
- As the measure of curl decreases, the degree to which they are "curled up" increases.

$$\text{fibre length} = \frac{\text{perimeter-}\sqrt{(\text{perimeter})^2 - 16 \cdot \text{area}}}{4}$$

$$\text{fibre width} = \frac{\text{area}}{\text{fibre length}}$$

Convex Hull

- The convex hull of an object is defined to be the smallest convex shape that contains the object.



Solidity

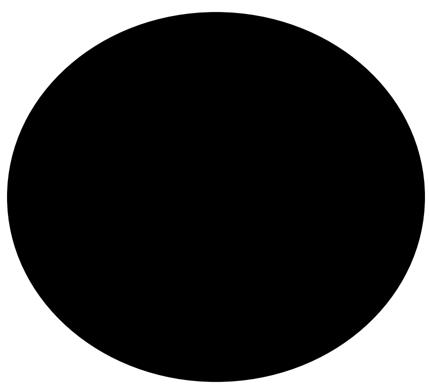
- Solidity measures the density of an object.

- A measure of solidity can be obtained as the ratio of the area of an object to the area of a convex hull of the object:

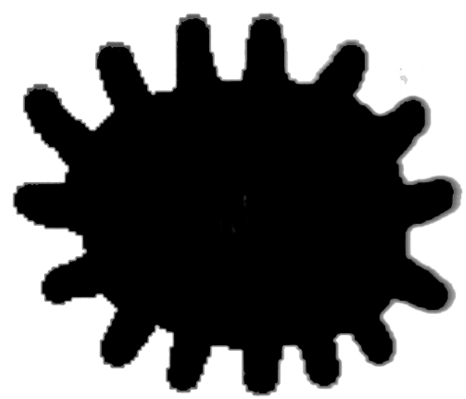
$$\text{solidity} = \frac{\text{area}}{\text{convex area}}$$

Solidity

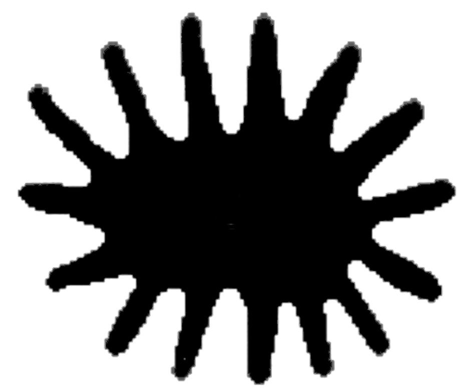
– A value of 1 signifies a solid object, and a value less than 1 will signify an object having an irregular boundary, or containing holes.



solidity=1.0



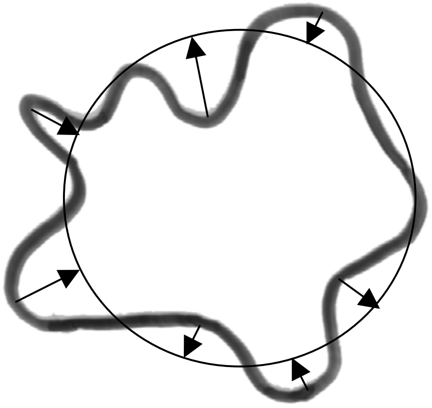
solidity=0.782



solidity=0.592

Shape Variances

- Sometimes a shape should be compared against a template.
  - A circle is an obvious and general template choice. The circular variance is the proportional mean-squared error with respect to solid circle.
  - It gives zero for a perfect circle and increases along shape complexity and elongation.



Shape Variances

- Elliptic variance is defined similarly to the circular variance. An ellipse is fitted to the shape (instead of a circle) and the mean-squared error is measured.

Rectangularity

- Rectangularity is the ratio of the object to the area of the minimum bounding rectangle.
  – Let F_k be the ratio of region area and the area of a bounding rectangle, the rectangle having the direction k. The rectangle direction is turned in discrete steps as before, and rectangularity measured as a maximum of this ratio F_k

$$\text{rectangularity} = \max_{k}(F_k)$$

  – Rectangularity has a value of 1 for perfectly rectangular object

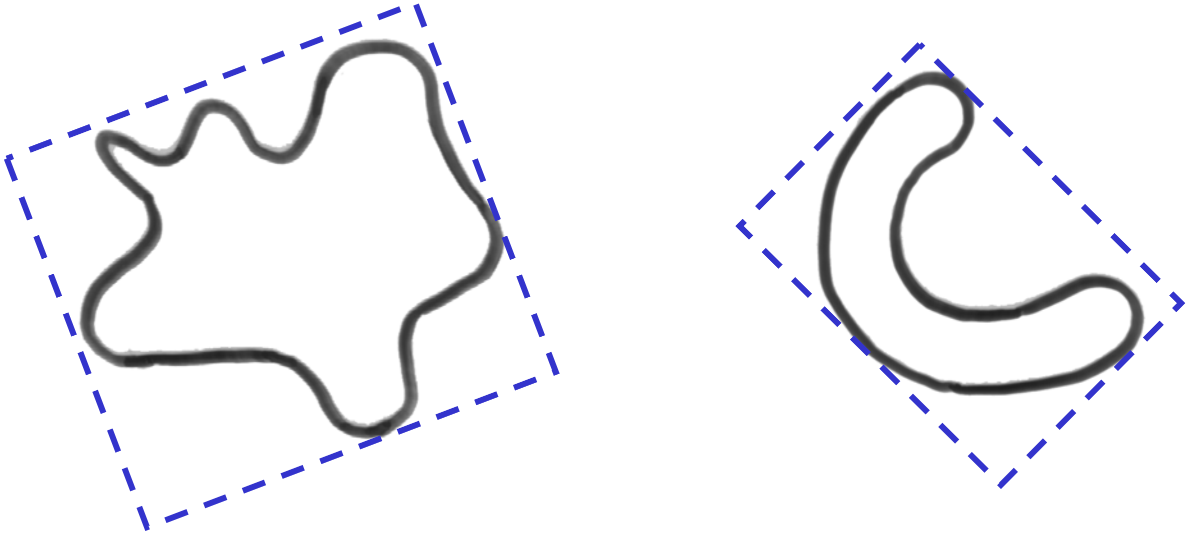
Bounding Box

- The bounding box or bounding rectangle of an object is a rectangle which circumscribes the object. The dimensions of the bounding box are those of the major and minor axes.
  - The area of the bounding box is:

$$\text{area} = (\text{major axis length}) * (\text{minor axis length})$$

  - The minimum bounding box is the minimum area that bounds the shape.

Bounding Box



bounding boxes

Direction

- Direction is a property which makes sense in elongated regions only. If the region is elongated, direction is the direction of the longer side of a minimum bounding rectangle.
  - If the shape moments are known, the direction θ can be computed as:

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{2\mu_{11}}{\mu_{20} - \mu_{02}}\right)$$

Direction

- Elongatedness and rectangularity are independent of linear transformations translation, rotation, and scaling.
- Direction is independent on all linear transformations which do not include rotation.
- Mutual direction of two rotating objects is rotation invariant.

Orientation

- The overall direction of the shape.

Topological Descriptors

- Topological properties are useful for global descriptions of objects in an image.
  - Features that do not change with elastic deformation of the object.
  - For binary regions, topological features include the number of holes in a region, and the number of indentations, or protrusions.
  - One topological property is the number of connected components.

Topological Descriptors

- The number of holes **H** and connected components **C** in an image can be used to define the Euler number.
  - The Euler number is defined as the number of components minus the number of holes:

$$\text{Euler number} = \mathbf{C} - \mathbf{H}$$

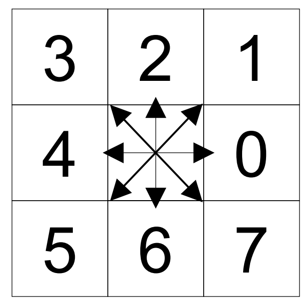
  - This simple topological feature is invariant to translation, rotation and scaling.

Boundary Descriptors

- The shape of a region can be represented by quantifying the relative position of consecutive points on its boundary.
- A chain code consists of a starting location and a list of directions $d_1, d_2, \ldots, d_N$ provides a compact representation of all the information in a boundary.
  - The directions d_i are estimates of the slope of the boundary.

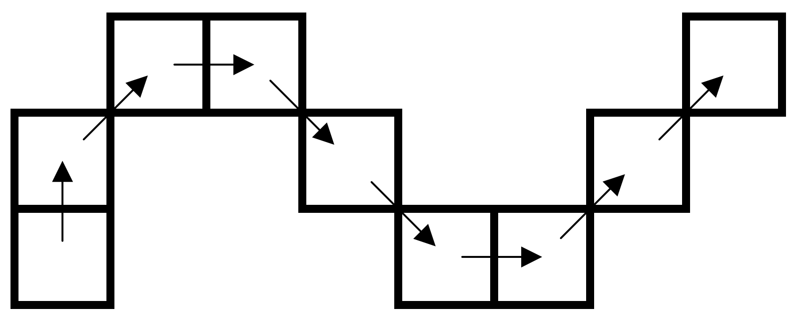
Boundary Descriptors

- Chain codes are based on 4- or 8-connectivity:



e.g.

2,1,0,7,7,0,1,1



Boundary Descriptors

- The *k-slope* of the boundary at (x_i,y_i) can be estimated from the slope of the line joining $(x_{i-k/2},y_{i-k/2})$ and $(x_{i+k/2},y_{i+k/2})$ for some small, even value of *k*. Calculated as an angle of:

$$\tan^{-1}\left(\frac{y_{i+k/2} - y_{i-k/2}}{x_{i+k/2} - x_{i-k/2}}\right)$$

measured in a clockwise direction, with a horizontal slope taken to be zero.

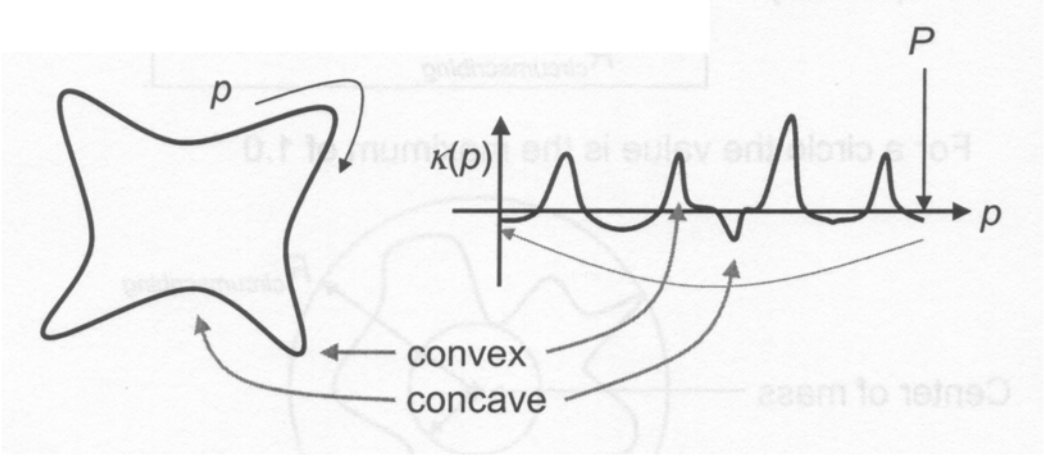
Boundary Descriptors: Curvature

- The *k-curvature* of the boundary at (x_i,y_i) can be estimated from the change in the *k-slope*:

$$\left\{\tan^{-1}\left(\frac{y_{i+k}-y_i}{x_{i+k}-x_i}\right)-\tan^{-1}\left(\frac{y_i-y_{i-k}}{x_i-x_{i-k}}\right)\right\} \pmod{2\pi}$$

Boundary Descriptors: Curvature

- The curvature (κ) of an object is a local shape attribute.



  - Convex shapes yield positive curvatures
  - Concave shapes yield negative curvatures

Boundary Descriptors: Bending Energy

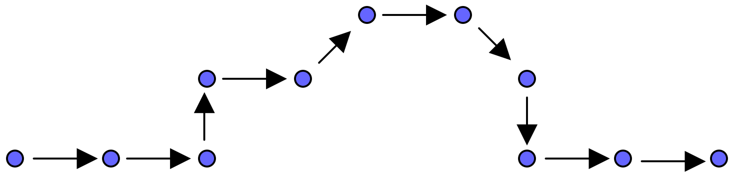
- The total bending energy E_C is a robust global shape descriptor.
  - The bending energy of a boundary may be understood as the energy necessary to bend a rod to the desired shape and can be calculated as a sum-of-squares of the boundary curvature $\kappa(p)$ over the boundary length L.

$$E_c = \frac{1}{L}\sum_{p=1}^{L} \kappa(p)^2 \qquad \frac{2\pi}{R} \le E_c \le \infty$$

  - The minimum value $2\pi/R$ is obtained for a circle of radius R.

Boundary Descriptors: Bending Energy

- For example:



chain-code: 0 0 2 0 1 0 7 6 0 0

curvature: 0 2 −2 1 −1 −1 −1 2 0

sum of squares: 0 4 4 1 1 1 1 4 0

Bending Energy = 16

Boundary Descriptors:

Total Absolute Curvature

- This descriptor is a measure of the total absolute curvature in an object:

$$\kappa_{total} = \frac{1}{L}\sum_{p=1}^{L}\left|\kappa(p)\right| \quad 2\pi \le \kappa_{total} \le \infty$$

  - The minimum value is found for all convex objects.

Moment Analysis

- The evaluation of moments represents a systematic method of shape analysis.
  - The most commonly used region attributes are calculated from the three low-order moments.
  - Knowledge of the low-order moments allows the calculation of the central moments, normalised central moments, and moment invariants.

Spatial Moments

- To define the $(p,q)^{th}$-order moment:

$$m_{pq} = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} x^p y^q \quad \text{for} \quad p,q = 0,1,2,\ldots$$

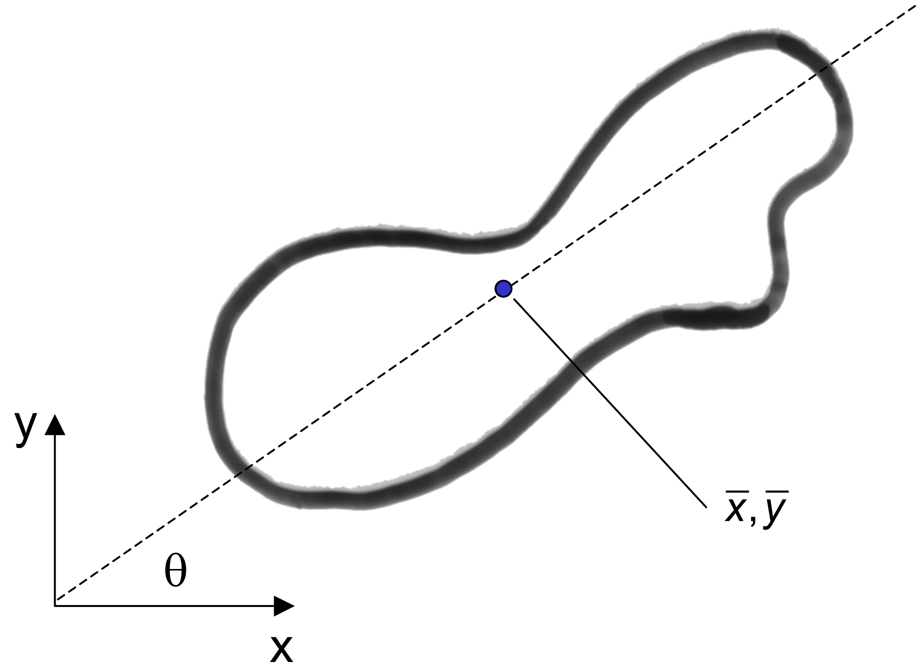
- The zeroth-order moment m_{00} simply represents the sum of the pixels contained in an object and gives a measure of the area (because $x^0=y^0=1$)

Spatial Moments

- The first-order moments in x (m_{10}) and y (m_{01}) normalised by the area can be used to specify the location of an object:
  - The *centre of gravity*, or *centroid* of an object is a measure of the object's location in the image.
  - It has two components, denoting the row and column positions of the point of balance of the object.

$$\text{centroid} = \left(\bar{x}, \bar{y}\right) = \left(\frac{m_{10}}{m_{00}}, \frac{m_{01}}{m_{00}}\right)$$

Spatial Moments



Central Moments

- The central moments μ_{pq} (i.e. $p+q>1$) represent descriptors of a region that are normalised with respect to location.

$$\mu_{pq} = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left(x-\bar{x}\right)^{p}\left(y-\bar{y}\right)^{q} \quad \text{for } p+q>1$$

Normalised Central Moments

- The central moments can be normalised with respect to the zeroth moment to yield the normalised central moment:

$$\eta_{pq} = \mu_{pq} / \mu_{00}^{\gamma}$$

$$\gamma = (p+q)/2+1$$

- The most commonly normalised central moment is η_{11}, the first central moment in x and y.
  - This provides a measure of the deviation from a circular region shape. A value close to zero describes a region that is close to circular.

Central Moments

- Central moments are translation invariant:
  - i.e. two objects that are identical except for having different centroids, will have identical values of μ_{pq}
  - Central moments are not rotationally invariant
- Central moments are not rotationally invariant – they will change if an object is rotated.

Central Moments

- The second-order central moments:

$$\mu_{20} = m_{20} - \frac{m_{10}^2}{m_{00}} \quad \mu_{02} = m_{02} - \frac{m_{01}^2}{m_{00}} \quad \mu_{11} = m_{11} - \frac{m_{10}m_{01}}{m_{00}}$$

- The second-order moments measure how dispersed the pixels in an object are from their centroid:
  - μ_{20} measures the object's spread over rows
  - μ_{02} measures the object's spread over columns
  - μ_{11} is a cross-product term representing spread in the direction in which both row and column indices increase.

Principal Axes

- Principal axes of an object can be uniquely defined as segments of lines crossing each other orthogonally in the centroid of the object and representing the directions with zero cross-correlation. This way, a contour is seen as an realization of a statistical distribution.

Principal Axes

- Principal (major and minor) axes are defined to be those axes that pass through the centroid, about which the moment of inertia of the region is, respectively maximal or minimal.
  - The orientation of the major axis:

$$\theta = \frac{1}{2}\tan^{-1}\left[\frac{2\mu_{11}}{\mu_{20} - \mu_{02}}\right]$$

    which is measured clockwise, with the horizontal direction taken as zero.
  - This can be used to find the minimum bounding box.

Moment Invariants

- Normalisation with respect to orientation results in rotationally invariant moments.
  - The first two are the following functions of the second-order central moments:

$$\phi_1 = \eta_{20} + \eta_{02}$$

$$\phi_2 = \left(\eta_{20} - \eta_{02}\right)^2 + 4\eta_{11}^2$$

  - The first of these statistics is the moment of inertia, a measure of how dispersed, in any direction, the pixels in an object are from their centroid.
  - The second statistic measures whether this dispersion is isotropic or directional.

Moment Invariants

$$\phi_1 = \eta_{20} + \eta_{02}$$

$$\phi_2 = \left(\eta_{20} - \eta_{02}\right)^2 + 4\eta_{11}^2$$

$$\phi_3 = \left(\eta_{30} - 3\eta_{12}\right)^2 + \left(3\eta_{21} - \eta_{03}\right)^2$$

$$\phi_4 = \left(\eta_{03} + \eta_{12}\right)^2 + \left(\eta_{21} + \eta_{03}\right)^2$$

Central Moments

$$\mu_{10} = \mu_{01} = 0$$

$$\mu_{11} = m_{11} - m_{10} m_{01} / m_{00}$$

$$\mu_{20} = m_{20} - m_{10}^2 / m_{00}$$

$$\mu_{02} = m_{02} - m_{01}^2 / m_{00}$$

$$\mu_{30} = m_{30} - 3\bar{x} m_{20} + 2 m_{10} \bar{x}^2$$

$$\mu_{03} = m_{03} - 3\bar{y} m_{02} + 2 m_{01} \bar{y}^2$$

$$\mu_{12} = m_{12} - 2\bar{y} m_{11} - \bar{x} m_{02} + 2 m_{10} \bar{y}^2$$

$$\mu_{21} = m_{21} - 2\bar{x} m_{11} - \bar{y} m_{20} + 2 m_{01} \bar{x}^2$$

References:

Moments

1. Pohlman, K.A., Powell, K.A., Obuchowski, N.A., Chilcote, W.A., Grunfest-Bronlatowski, S., “Quantitative classification of breast tumours in digitized mammograms”, *Medical Physics,* 1996, **23**: pp.1337-1345 **(masses)**
2. Rangayyan, R.M., El-Faramawy, N.M., Desautels, J.E.L., Alim, O.A., “Measures of acutance and shape classification of breast tumors”, *IEEE Transactions on Medical Imaging,* 1997, **16**: pp.799-810 **(masses)**
3. Shen, L., Rangayyan, R.M., and Desautels, J.E.L., “Application of shape analysis to mammographic calcifications”, *IEEE Transactions on Medical Imaging,* 1994, **13**: pp.263-274 **(calcifications)**
4. Wei, L., Jianhong, X., Micheli-Tzanakou, E., “A computational intelligence system for cell classification”, *Proceedings of the IEEE International Conference on Information Technology Applications in Biomedicine*, 1998, pp.105-109 **(blood cells)**

Radial Distance Measures

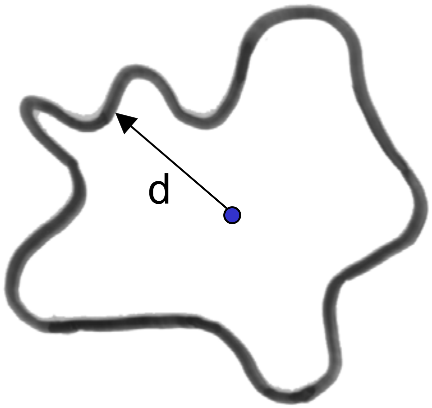
- The shape of a structure of interest can be determined by analysing its boundary, the variations and curvature of which constitute the information to be quantified.
  - Transform the boundary into a 1D signal and analysing its structure.
  - The radial distance is measured from the central point (centroid) in the object to each pixel x(n), y(n) on the boundary.

Radial Distance Measures

- Generally the centroid is used as the central point, and the radial distance:

$$d(n) = \sqrt{[x(n) - \bar{x}]^2 + [y(n) - \bar{y}]^2} \qquad n = 0, 1, \ldots, N-1$$

is obtained by tracing all *N* pixels of the boundary.



Radial Distance Measures

- To achieve scale invariance, the normalised radial distance $r(n)$ is obtained by normalising $d(n)$ with the maximal distance
- The number of times the signal $r(n)$ crosses its mean and other similar metrics can be used as a measure of boundary roughness.
  - Kilday, J., Palmieri, F., and Fox, M.D., "Classifying mammographic lesions using computerized image analysis", *IEEE Transactions on Medical Imaging,* 1993, **12**: pp.664-669

Radial Distance Measures

- The sequence $r(n)$ is further analysed to extract shape metrics such as the entropy:

$$\mathrm{E} = \sum_{k=1}^{K} h_k \log h_k$$

where h_k is the k-bin probability histogram that represents the distribution of $r(n)$ as well as the statistical moments:

$$m_p = \frac{1}{N} \sum_{n=0}^{N-1} \left[r(n) \right]^p$$

Radial Distance Measures

- The central moment:

$$\mu_p = \frac{1}{N}\sum_{n=0}^{N-1}\left[r(n) - m_1\right]^p$$

- Normalised moments invariant to translation, rotation and scaling:

$$\bar{m}_p = \frac{m_p}{\mu_2^{p/2}} \qquad \bar{\mu}_p = \frac{\mu_p}{\mu_2^{p/2}} \qquad p \neq 2$$

Fourier Descriptor

- The information in the *r*(*n*) signal can be further analysed in the spectral domain using the discrete Fourier transform (DFT):

$$a(u) = \frac{1}{N}\sum_{n=0}^{N-1} r(n)e^{-j2\pi nu/N} \qquad u = 0,1,\ldots,N-1$$

  – The “low-frequency” terms, correspond to the smooth behavior.
  – The “high-frequency” terms correspond to the jagged, bumpy behavior → roughness

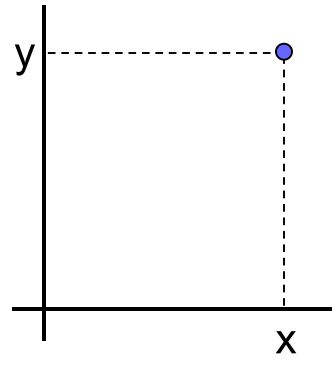
References:

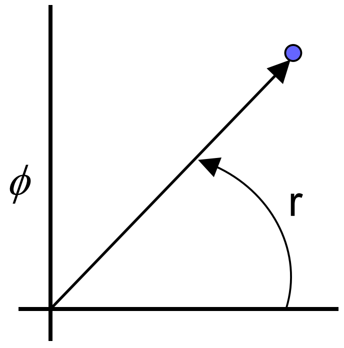
Radial Distance Measures

1. Pohlman, K.A., Powell, K.A., Obuchowski, N.A., Chilcote, W.A., Grunfest-Bronlatowski, S., “Quantitative classification of breast tumours in digitized mammograms”, *Medical Physics,* 1996, **23**: pp.1337-1345 **(masses)**
2. Bruce, L.M., Kallergi, M., “Effects of image resolution and segmentation method on automated mammographic mass shape classification”, *Proceedings of the SPIE*, 1999, **3661**: pp.940-947 **(masses)**
3. Kilday, J., Palmieri, F., and Fox, M.D., “Classifying mammographic lesions using computerized image analysis”, *IEEE Transactions on Medical Imaging,* 1993, **12**: pp.664-669 **(masses)**
4. Shen, L., Rangayyan, R.M., and Desautels, J.E.L., “Application of shape analysis to mammographic calcifications”, *IEEE Transactions on Medical Imaging,* 1994, **13**: pp.263-274 **(calcifications)**

Contour-based Shape Representation & Description

- Object boundaries must be expressed in some mathematical form:
  - Rectangular (Cartesian) coordinates
  - Polar coordinates (in which boundary elements are represented as pairs of angle ϕ and distance *r*.





Fractal Dimension

- The fractal dimension is the rate at which the perimeter of an object increases as the measurement scale is reduced.
  - The fractal dimension produces a single numeric value that summarizes the irregularity of “roughness” of the feature boundary

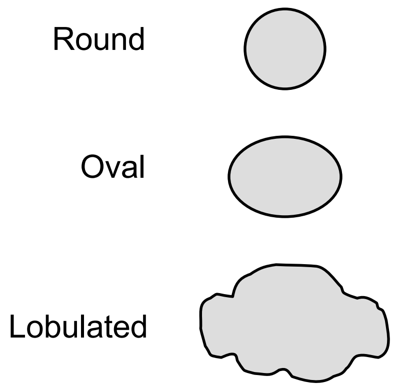
Fractal Dimension

- Richardson dimension
  - Counting the number of strides needed to “walk” along the boundary, as a function of stride length.

    number of steps $\times$ stride-length = perimeter measurement
  - As the stride length is reduced, the path follows more of the local irregularities of the boundary and the measured parameter increases.
  - The result, plotted on log-log axes is a straight line whose slope gives the fractal dimension.

Mammogram Features

- Shape descriptors can be used to characterise features extracted from mammograms.



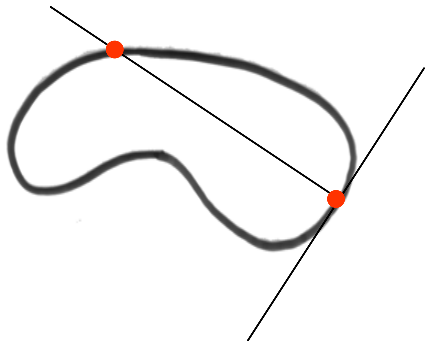
Cell Image Features

- When dealing with images containing numerous objects, such as histological or cytological cell images, shape descriptors are calculated for all the individual cells.
  – Global shape measures can be calculated from the individual image descriptors:
    - Standard deviation → area, short-axis, long-axis, perimeter, circularity,
    - Mean → compactness, elongation, perimeter, area

Profile

- The profile of a binary image analyses the binary image in terms of its projections.
  - Projections can be vertical, horizontal, diagonal, circular, radial, spiral.

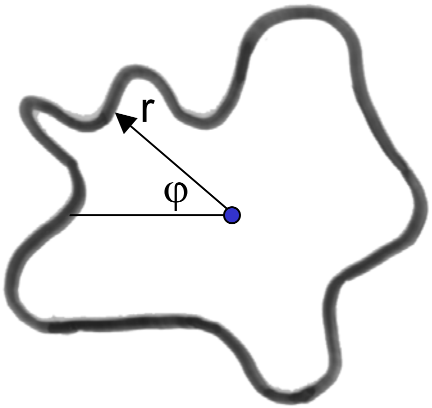
    e.g. used in determining a region of highest density



  - A profile is the sum of the pixel values in a specified direction

Signature Analysis

- A signature is a one-dimensional representation of the boundary.
  - Computing the distance from the centroid of an object to the boundary as a function of angle from 0°→360° in any chosen increment.
  - Harmonic analysis, or shape un
  - The plot repeats every 2π



Hough Transform

- The Hough transform is a global technique that finds the occurrence of objects of a predefined shape.
  - Used to identify shapes within a binary image containing disconnected points.
  - A cluster of such points may assume the shape of a line, a circle etc.
  - The central idea of the Hough transform is to represent a line made up of many pixels by a single peak in parametric space → the accumulator array

Hough Transform

– This single peak has coordinate values in the accumulator array of two parameters necessary to describe the line, such as slope and intersect.
– Permits the detection of parametric curves
  • e.g. circles, straight lines, ellipses, spheres, ellipsoids etc.

Hough Transform

- Consider the parametric representation of a line:

$$y = mx + c$$

  - In parameter space (m,c), any straight line in image space is represented by a single point.
  - Any line that passes through a point (p,q) in image space corresponds to the line c=–mp+q in parameter space.

Hough Transform

- To detect straight lines in an image:
  - Quantise the parameter space (m,c) and create an accumulator array (each dimension in the array corresponds to one of the parameters).
  - For every “1” pixel (x_i,y_i) in the binary image calculate $c=-mx_i+y_i$ for every value of the parameter m and increment the value of the entry (m,c) in the accumulator array by one.

Hough Transform

- This model is inadequate for representing vertical lines, a case for which m approaches infinity. To address this problem the normal representation of a line can be used:

$$r = x\cos\theta + y\sin\theta$$

- This equation describes a line having orientation θ at distance r from the origin. Here, a line passing through a point (x_i,y_i) in the image corresponds to a sinusoidal curve $r = x_i\cos\theta + y_i\sin\theta$

Hough Transform

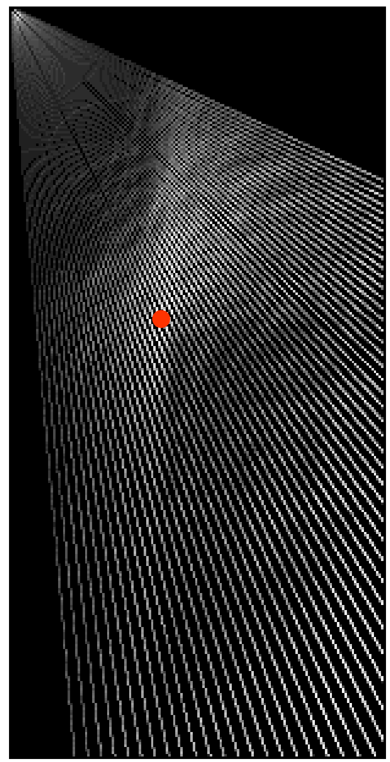
Algorithm:

1. Quantise the parameter space → accumulator array
2. Initialise all the cells in the accumulator array to zero.
3. For each point (x,y) in the image space, increment by 1 each of the accumulators that satisfy the equation.
4. Maxima in the accumulator array correspond to the parameters in the model.

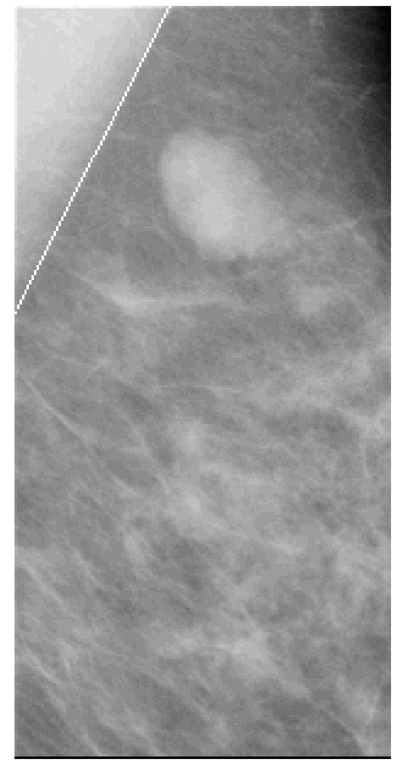
Hough Transform: Finding the Boundary of the Pectoral Muscle

binary image

accumulator array



extracted boundary



Terminology

- A line joining any two points on the boundary of an object is known as a chord.

Fourier Descriptors

- Staib, L.H., Duncan, J.S., “Boundary fitting with parametrically deformable models”, *IEEE Transactions on Pattern Analysis and Machine Intelligence,* 1992, **14**(11): pp.1061-1075