Grayscale Morphological Analysis

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Grayscale Morphology

- The elementary binary morphological operations can be extended to grayscale images through the use of **min** and **max** operations.
  - To perform morphological analysis on a grayscale image, regard the image as a height map.
  - **min** and **max** filters attribute to each image pixel a new value equal to the minimum or maximum value in a neighborhood around that pixel.
    - The neighborhood represents the shape of the structuring element.

Grayscale Morphology

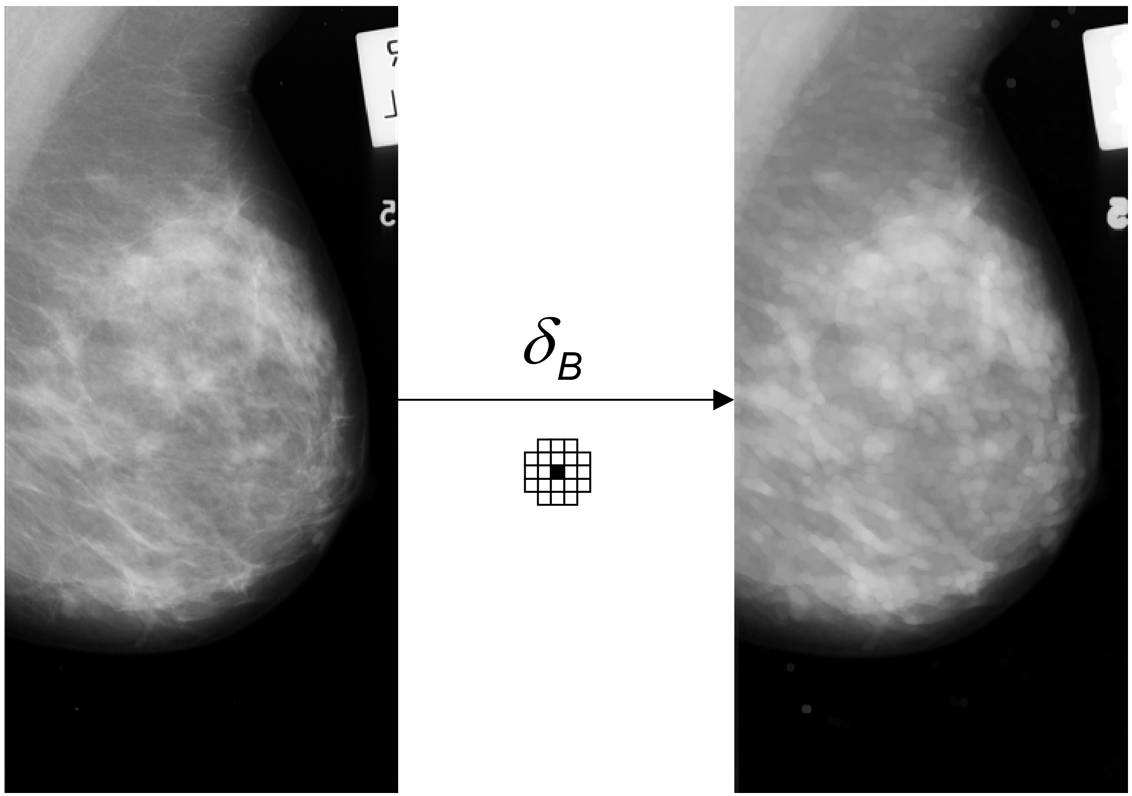
  - The **min** and **max** filters are analogous to erosion and dilation.
- Grayscale morphology has applications in:
  - contrast-enhancement
  - texture description
  - edge detection
  - thresholding

Grayscale Dilation

- The grayscale dilation of an image involves assigning to each pixel, the maximum value found over the neighborhood of the structuring element.
- The dilated value of a pixel *x* is the maximum value of the image in the neighborhood defined by the SE when its origin is at *x*:

$$\delta_B(f)_x = (f \oplus B)_x = \max_{\beta \in B} f(x + \beta)$$

Grayscale Dilation

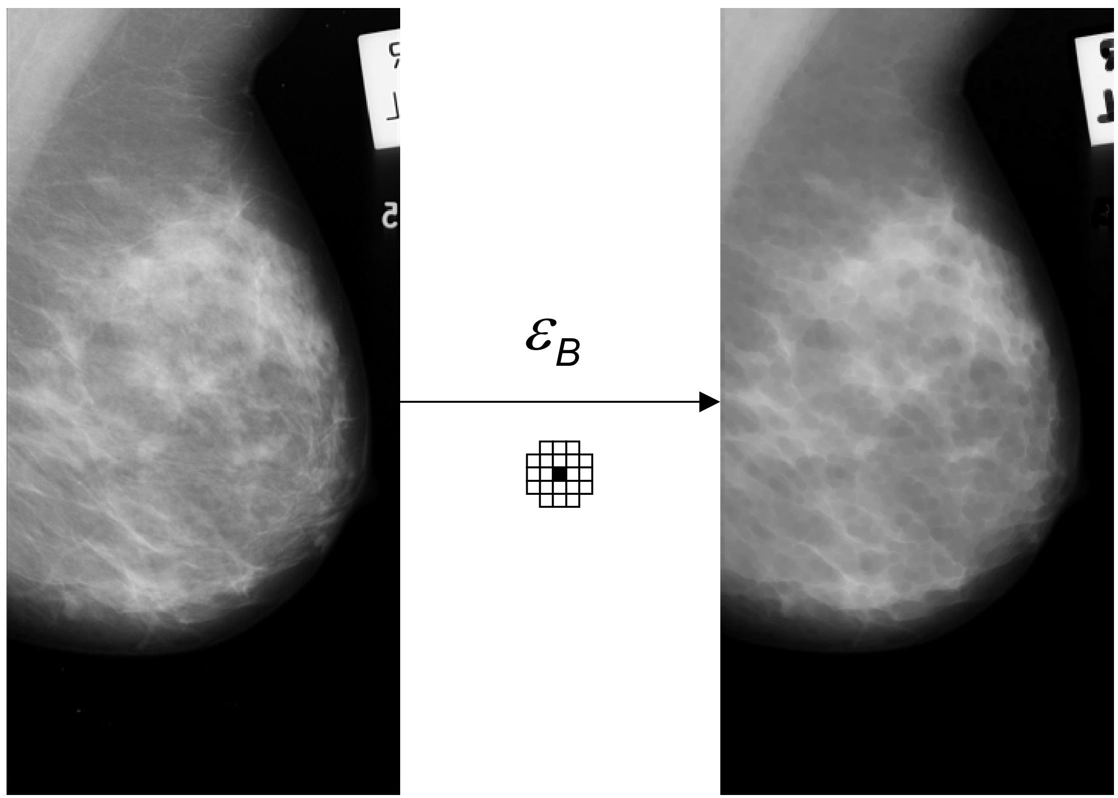


Grayscale Erosion

- The grayscale erosion of an image involves assigning to each pixel, the minimum value found over the neighborhood of the structuring element.
- The eroded value of a pixel *x* is the minimum value of the image in the neighborhood defined by the SE when its origin is at *x*:

$$\varepsilon_B(f)_x = (f \ominus B)_x = \min_{\beta \in B} f(x+\beta)$$

Grayscale Erosion

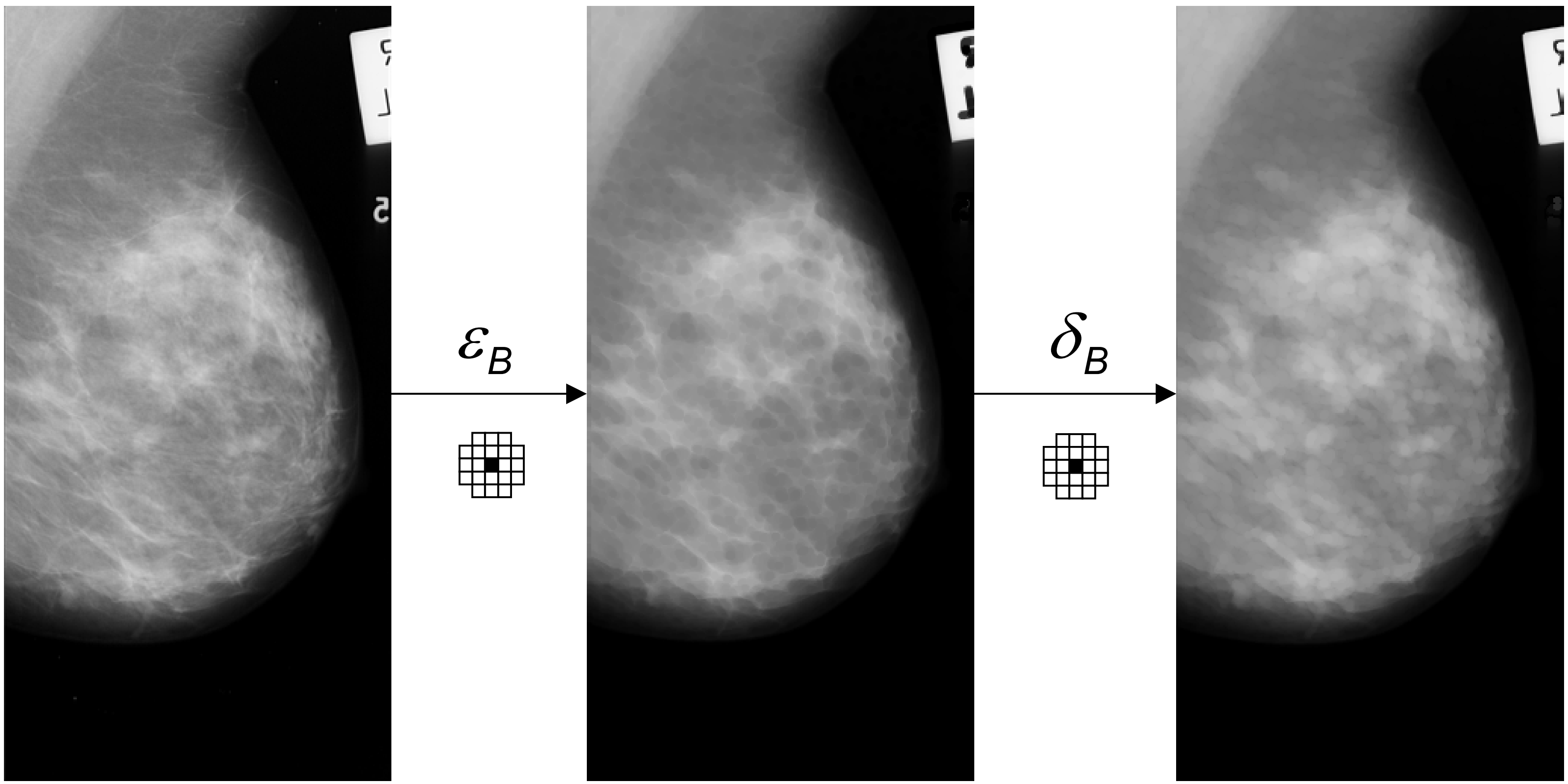


Grayscale Opening

- The grayscale opening of an image involves performing a grayscale erosion, followed by grayscale dilation.
- The opened value of a pixel is the maximum of the minimum value of the image in the neighborhood defined by the SE:

$$\gamma_S = \delta_B(\varepsilon_B)$$

Grayscale Opening

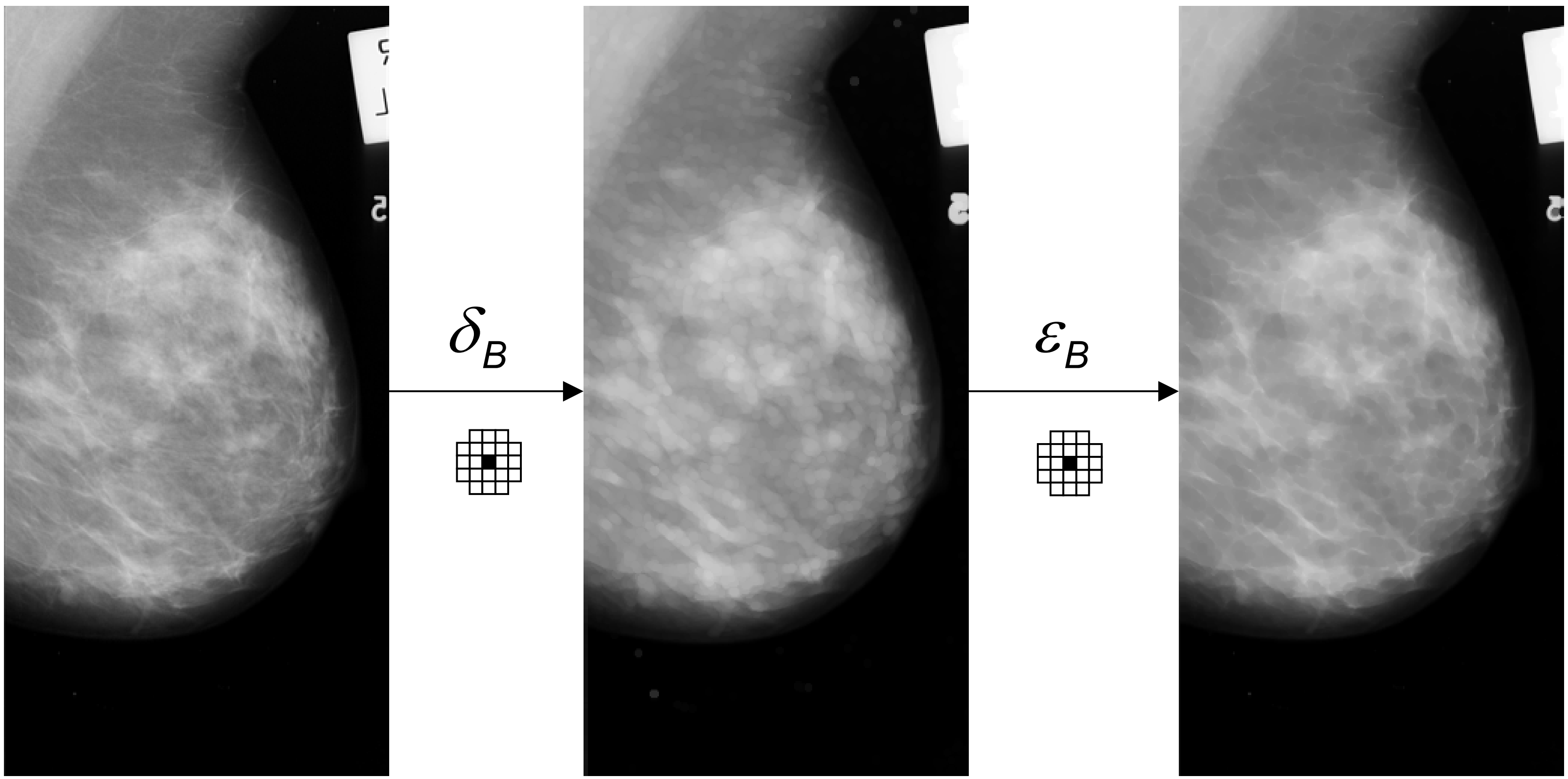


Grayscale Closing

- The grayscale closing of an image involves performing a grayscale dilation, followed by grayscale erosion.

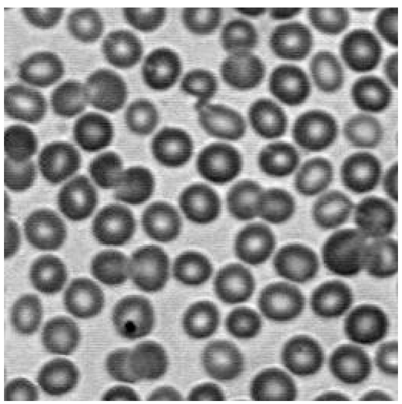
$$\phi_S = \varepsilon_B(\delta_B)$$

Grayscale Closing

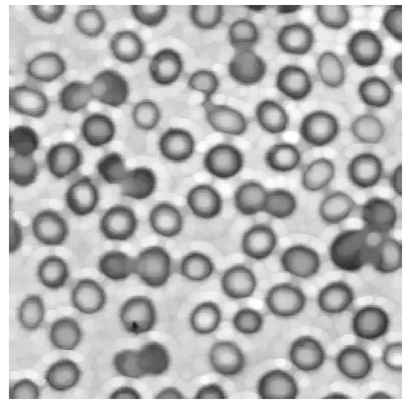


Overview Elementary Grayscale Morphological Operations

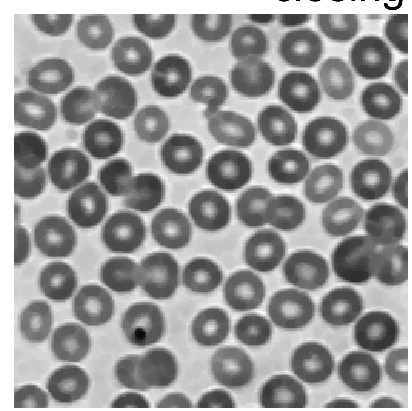
original



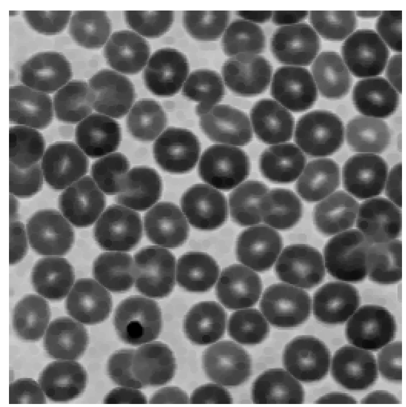
dilation



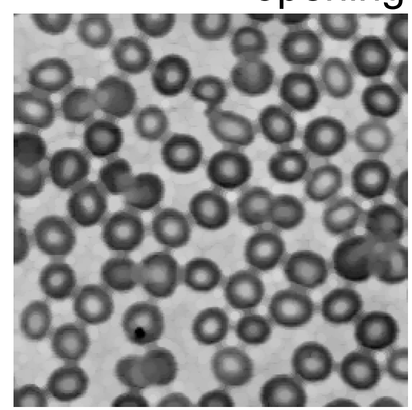
closing



erosion



opening



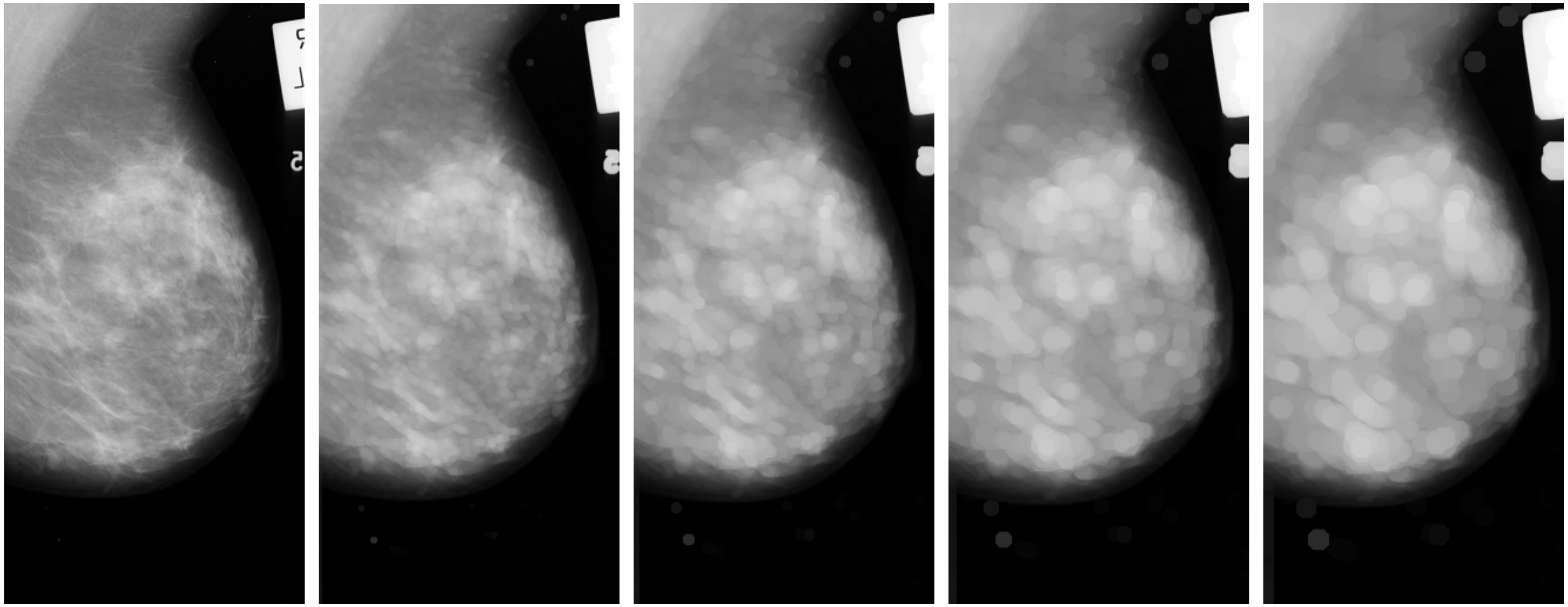
Depth

- The depth is the number of iterations of a particular operation.

Depth

e.g. Morphological Dilation at different depths, *d*

Original | *d*=1 | *d*=2 | *d*=3 | *d*=4

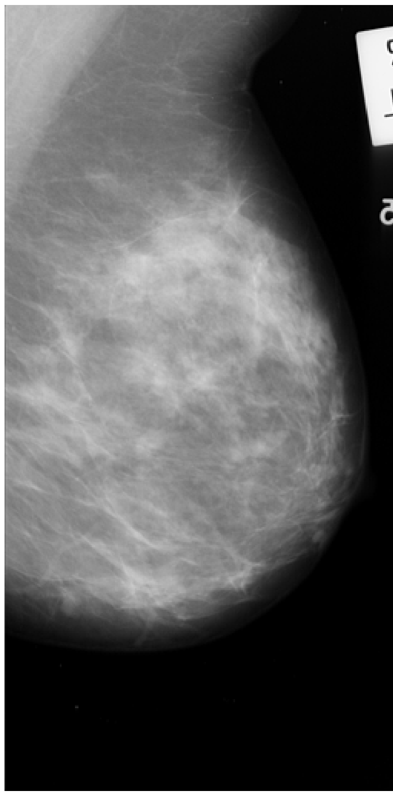


Morphological Smoothing

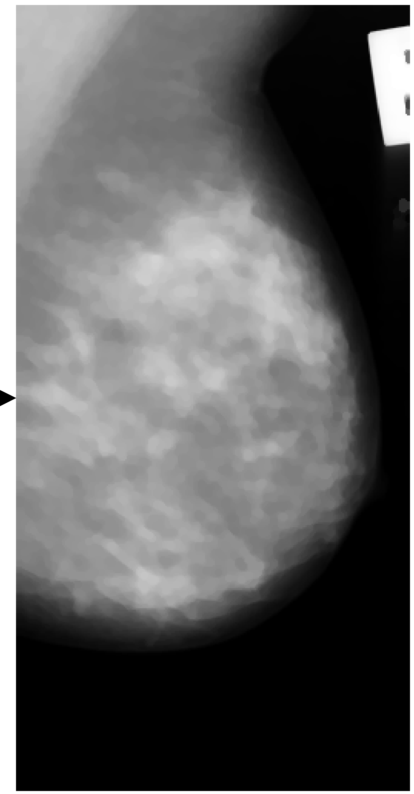
- A basic morphological smoothing is an opening followed by a closing operation.
  - It removes both bright and dark artifacts of noise.

$$\text{Smoothing} = \phi_S(\gamma_S)$$

Morphological Smoothing



$\phi_S(\gamma_S)$



Morphological Smoothing

- The average of the erosion and dilation of an image is analogous to image smoothing:

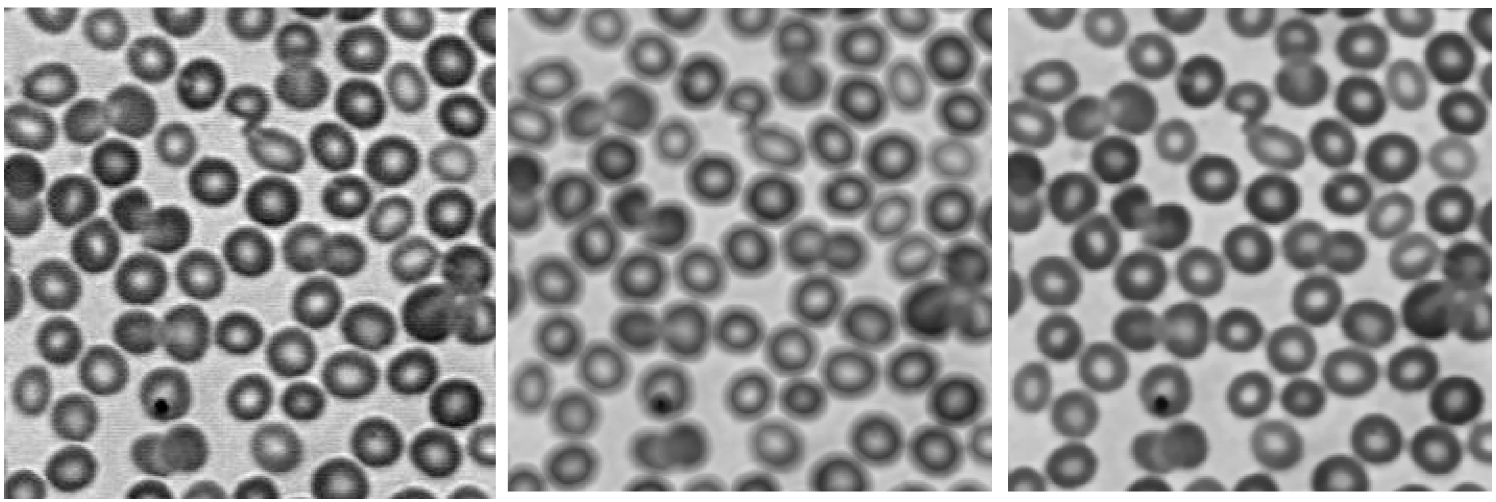
$$DYT(f) = \tfrac{1}{2}\left[\varepsilon_B(f) + \delta_B(f)\right]$$

- Texture smoothing:

$$TET(f) = \tfrac{1}{2}\left[\phi_B(f) + \gamma_B(f)\right]$$

Morphological Smoothing

DYT TET



Morphological Edge Detection

- Elementary grayscale morphological techniques can be used to distinguish smooth “ramp” edges, from ripple “texture” edges.
  - Non-ramp edges are texture or noise

Morphological Edge Detection

- Dynamic Lee Edge Detector:
  - Yields a result similar to the result from the linear Laplacian

$$DYL(f) = \text{point\_min}(\rho_B^+, \rho_B^-)$$

- Texture Lee

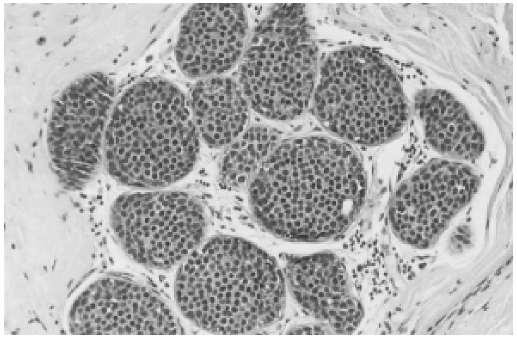
$$TEL(f) = \text{point\_min}(\phi - f, \gamma - f)$$

- Ramp Lee

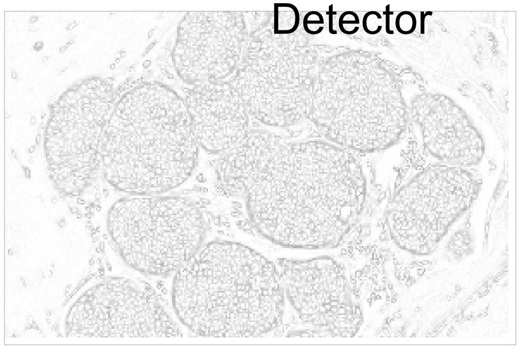
$$RAL(f) = \text{point\_min}(\delta - \phi, \gamma - \varepsilon)$$

Morphological Edge Detection

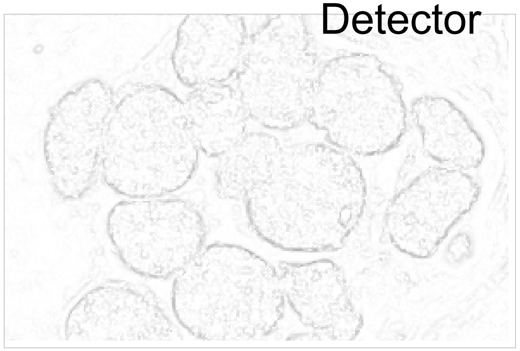
histological image:
breast carcinoma



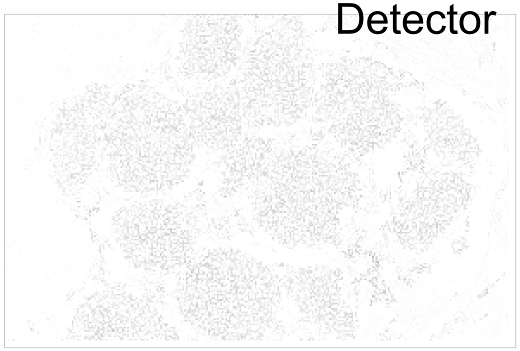
Lee Edge
Detector



Ramp-Lee Edge
Detector



Texture-Lee Edge
Detector



Morphological 2nd Derivatives

- Subtracting the morphologically smoothed image from the original image yields the morphological 2nd derivative image.
  - Isolates all edges.

$$DYG(f) = f - DYT(f)$$

Morphological 2nd Derivatives

- Isolate the non-ramp edges instead of all edges:

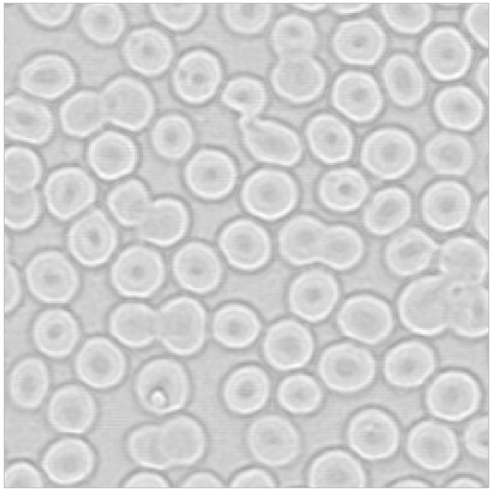
$$TEG(f) = f - TET(f)$$

- The difference between these two images represents the ramp edges, excluding texture or noise:

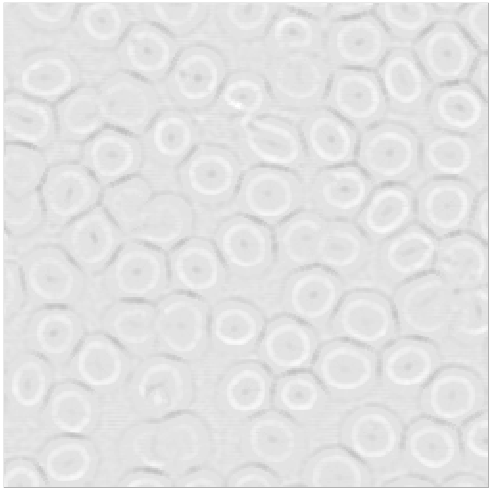
$$RAG(f) = DYG(f) - TEG(f)$$

Morphological 2^{nd} Derivatives

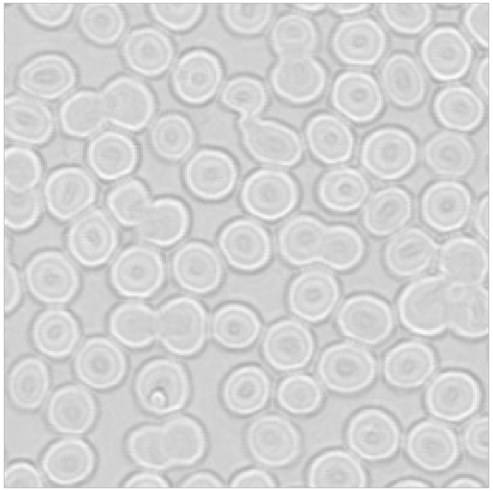
DYG



TEG

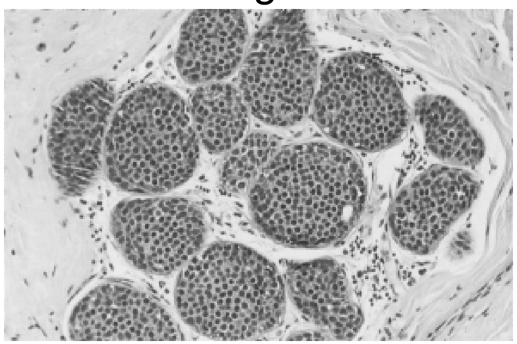


RAG

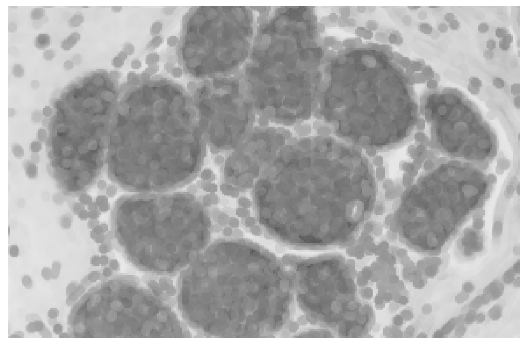


Morphological 2nd Derivatives

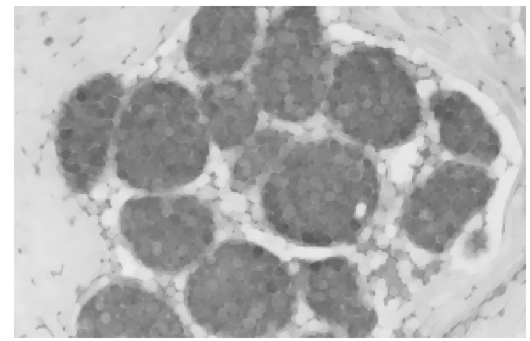
Original



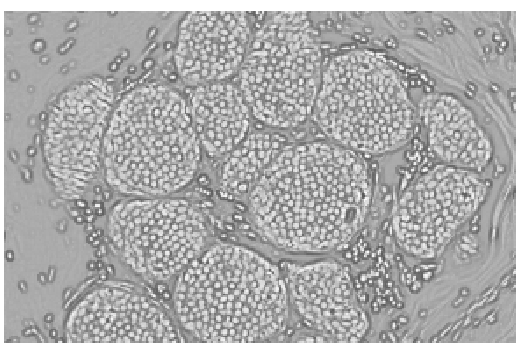
DYT



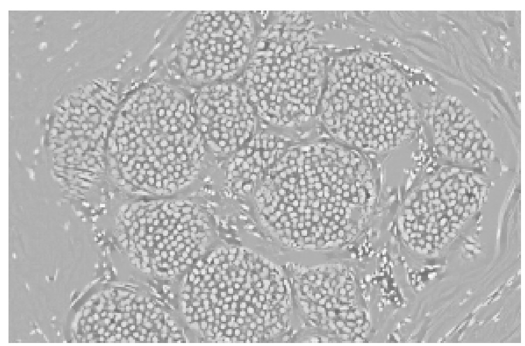
TET



DYG



TEG



RAG

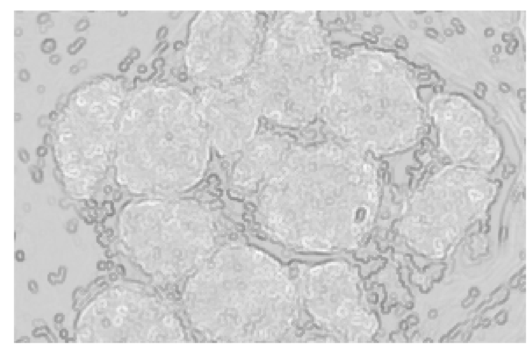
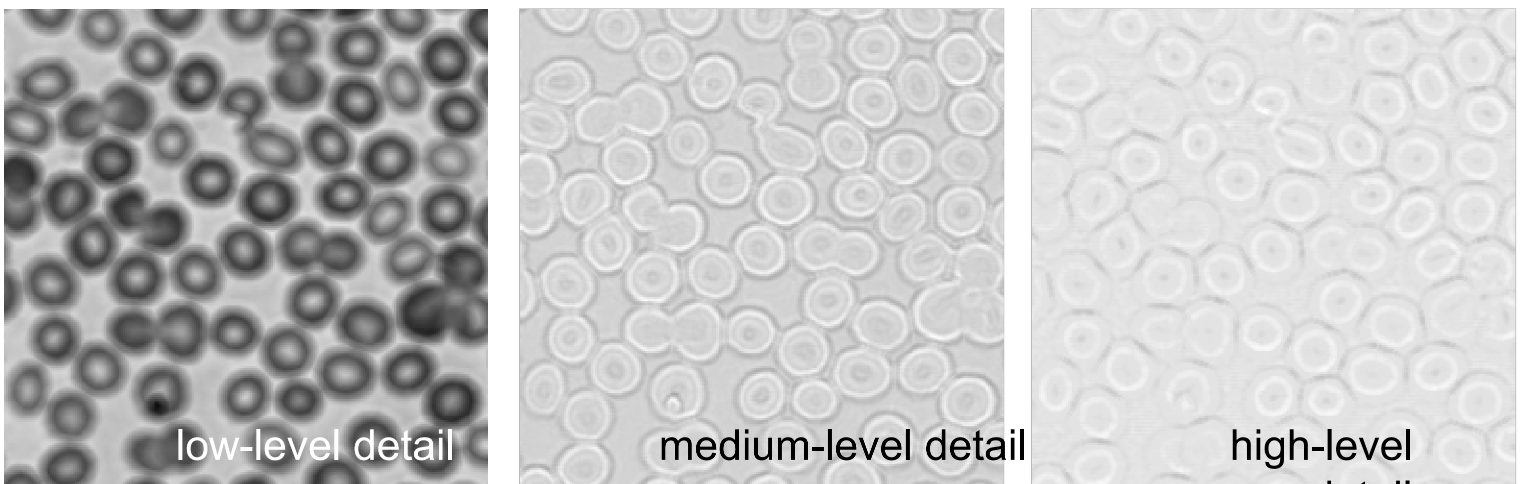


Image Details

- Consider the original image to be split up into low, medium and high detail contributions: *DYT*, *RAG*, and *TEG* respectively



Morphological Gradient

- A common assumption is that object boundaries or edges are located where there are high grayscale differences.
  - Gradient operators are used to enhance these variations.
  - If noise is present, the image should be filtered before applying a gradient operator so as to avoid enhancing the noise component.

Morphological Gradient

- A morphological gradient is the difference between a dilation and an erosion.
  - It highlights sharp gray level transitions.
  - Morphological gradients are operators enhancing variations of pixel intensity in a neighborhood determined by a SE.
  - Unlike gradients obtained using methods such as Sobel, morphological gradients obtained using symmetrical structuring elements tend to depend less on edge directionality.

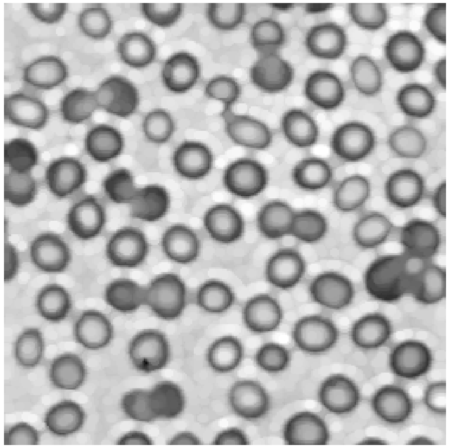
Basic Morphological Gradient

- The basic morphological gradient, also called the Beucher gradient, is defined as the arithmetic difference between the dilation and the erosion by the SE B.
  - This morphological gradient is denoted by ρ :

$$\rho_B = \delta_B - \varepsilon_B$$

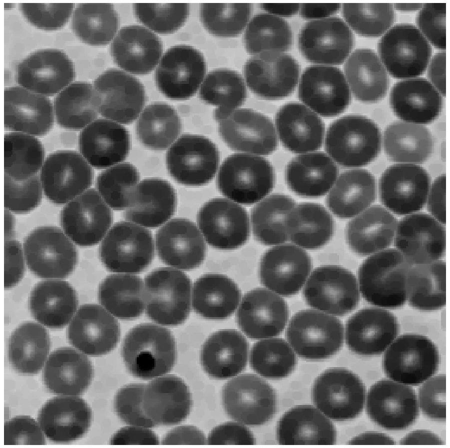
  - The morphological gradient returns the maximum variation (range) of the grayscale intensities within the neighborhood defined by the SE rather than a local slope.

Basic Morphological Gradient



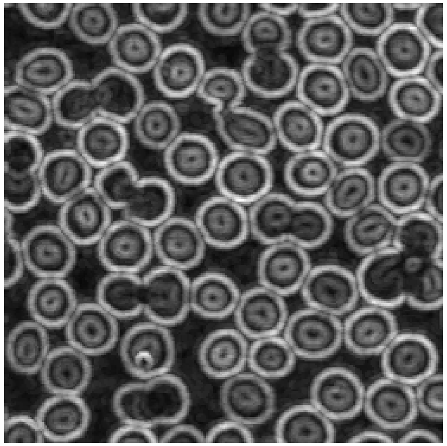
δ_B

$-$



ε_B

$=$



ρ_B

Morphological Gradient

- To calculate a gradient image which represents texture or noise:

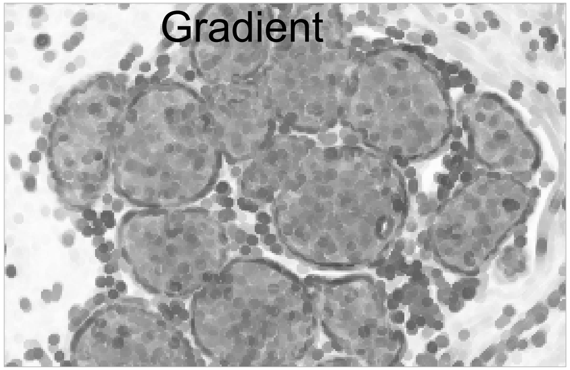
$$TER = \phi_B - \gamma_B$$

- To calculate a gradient image which represents only smooth edges:

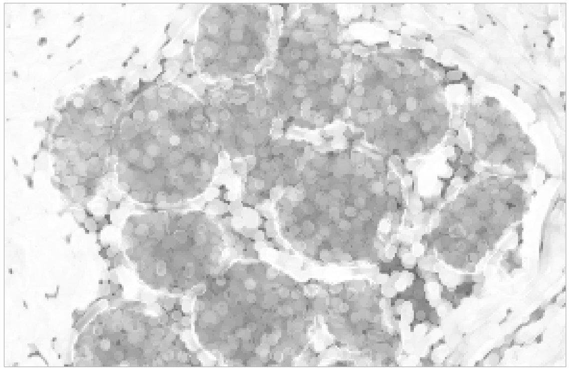
$$RAR = \rho_B - TER$$

Morphological Gradients

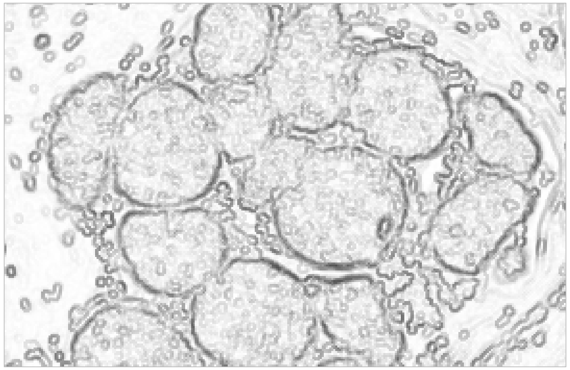
Morphological
Gradient



Texture Gradient (TER)



Ramp Gradient (RAR)



Half-Gradients

- The thickness of a step-edge detected by a morphological gradient equals two pixels:
  - one pixel on each side of the edge
- A zero-thickness can be achieved with inter-pixel approaches, or by defining the edge as the interface between two adjacent regions.
- Half-gradients can be used to detect either the internal or the external boundary of an edge.
  - These gradients are one-pixel thick for a step-edge

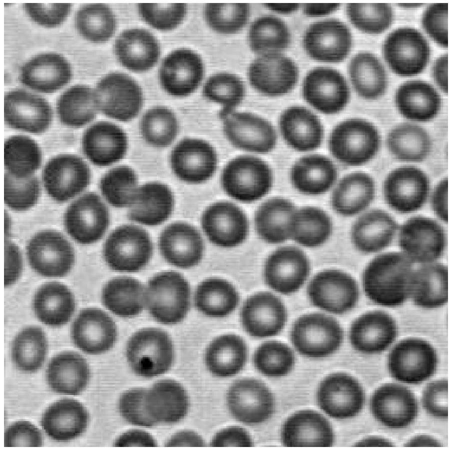
Half-Gradient by Erosion

- The half-gradient by erosion or internal gradient ρ^{-} is defined as the difference between the original image and the eroded image:

$$\rho_B^{-} = f - \varepsilon_B$$

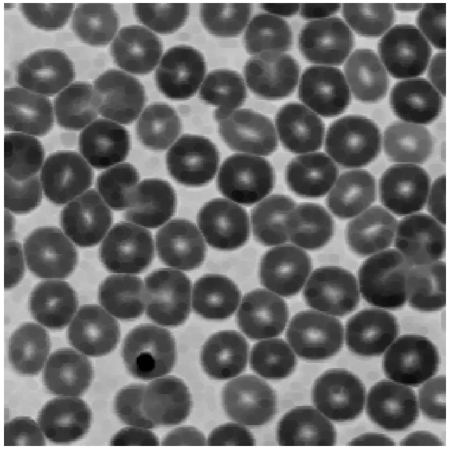
- The internal gradient enhances:
  - internal boundaries of objects brighter than their background, and
  - external boundaries of objects darker than their background.

Half-Gradient by Erosion



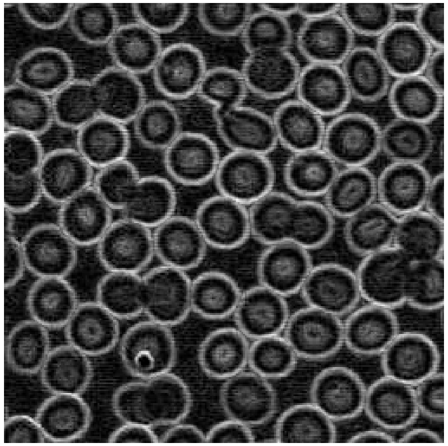
f

$-$



ε_B

$=$



ρ_B^-

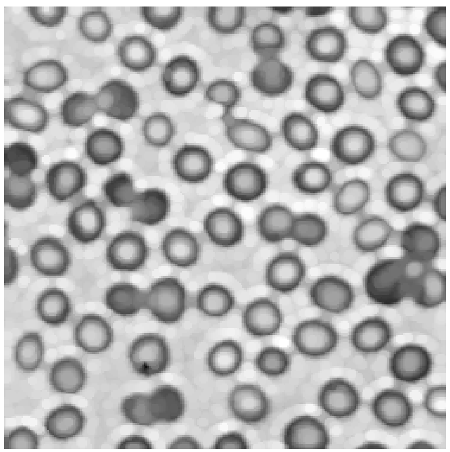
Half-Gradient by Dilation

- The half-gradient by dilation or external gradient ρ^{+} is defined as the difference between the dilated image and the original image:

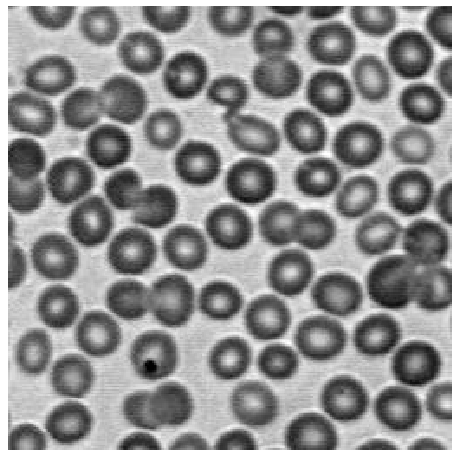
$$\rho_B^{+} = \delta_B - f$$

- The external gradient enhances:
  - internal boundaries of objects darker than their background, and
  - external boundaries of objects brighter than their background.

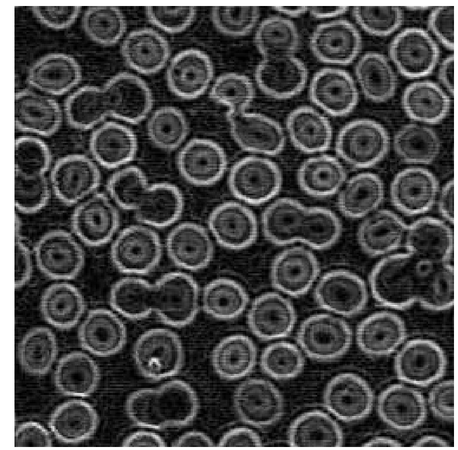
Half-Gradient by Dilation



$-$



$=$



δ_B f ρ_B^+

Half-Gradients

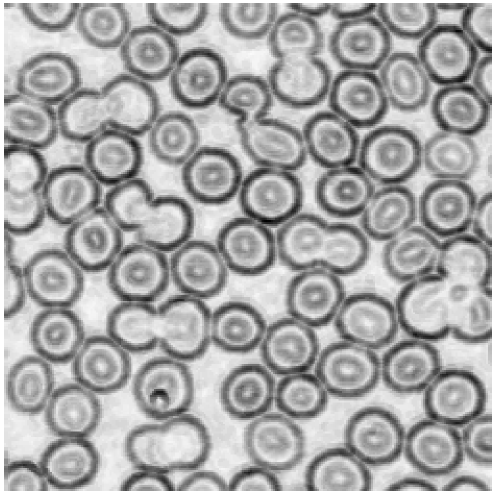
- The choice between internal and external gradients depends on the geometry and relative brightness of the structures to be extracted:

  e.g. a thin dark structure

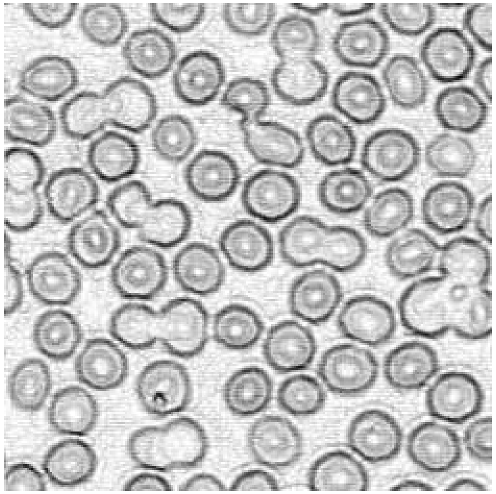
  external gradient → a thin-edge

  internal gradient → double edge

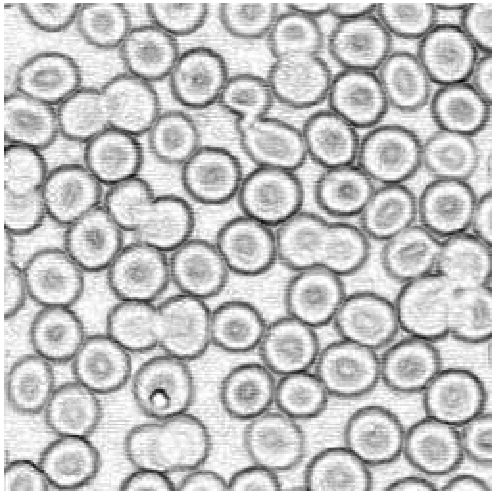
Comparing Half Gradients



ρ_B



ρ_B^+



ρ_B^-

Thick Gradients

- If the size of the SE is greater than 1, the morphological gradients are referred to as thick-gradients:

$$\rho_{nB} = \delta_{nB} - \varepsilon_{nB}$$

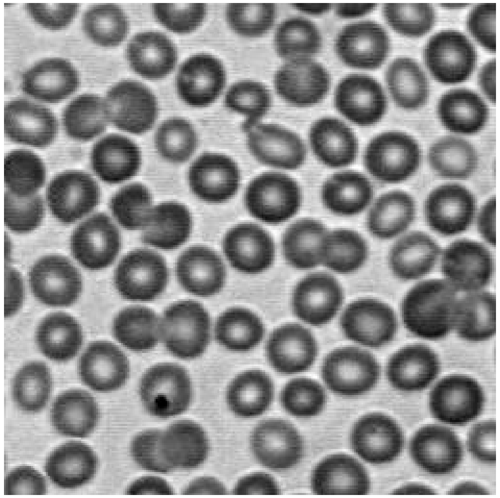
- Thick gradients give the maximum variation of the function in a neighborhood of size *n*.

Thick Gradients

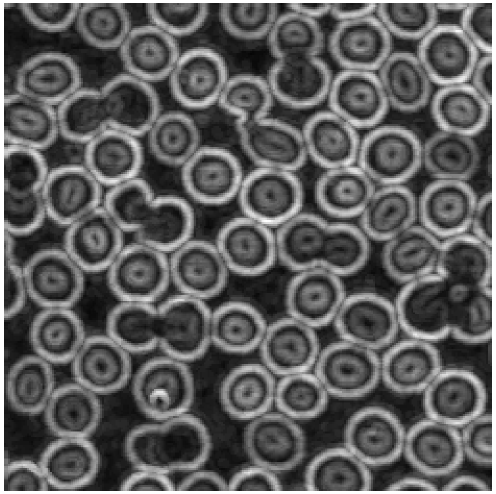
- If the size of *n* equals the width *e* of the transition between regions of homogeneous intensity, the thick-gradient will output the contrast value *h* (grayscale-difference), between these regions.
- Thick-gradients are recommended when transitions between objects are smooth.

Thick Gradients

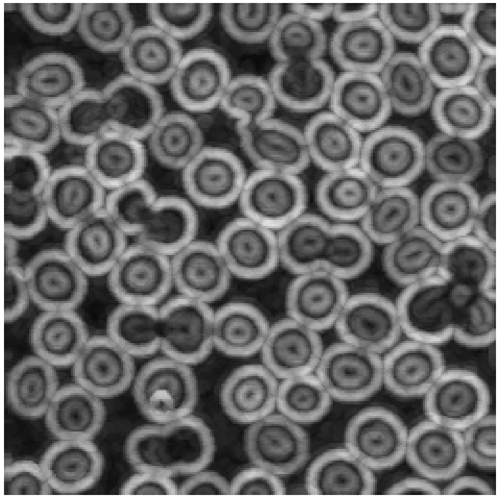
original



ρ_B



ρ_{nB}



Directional Gradients

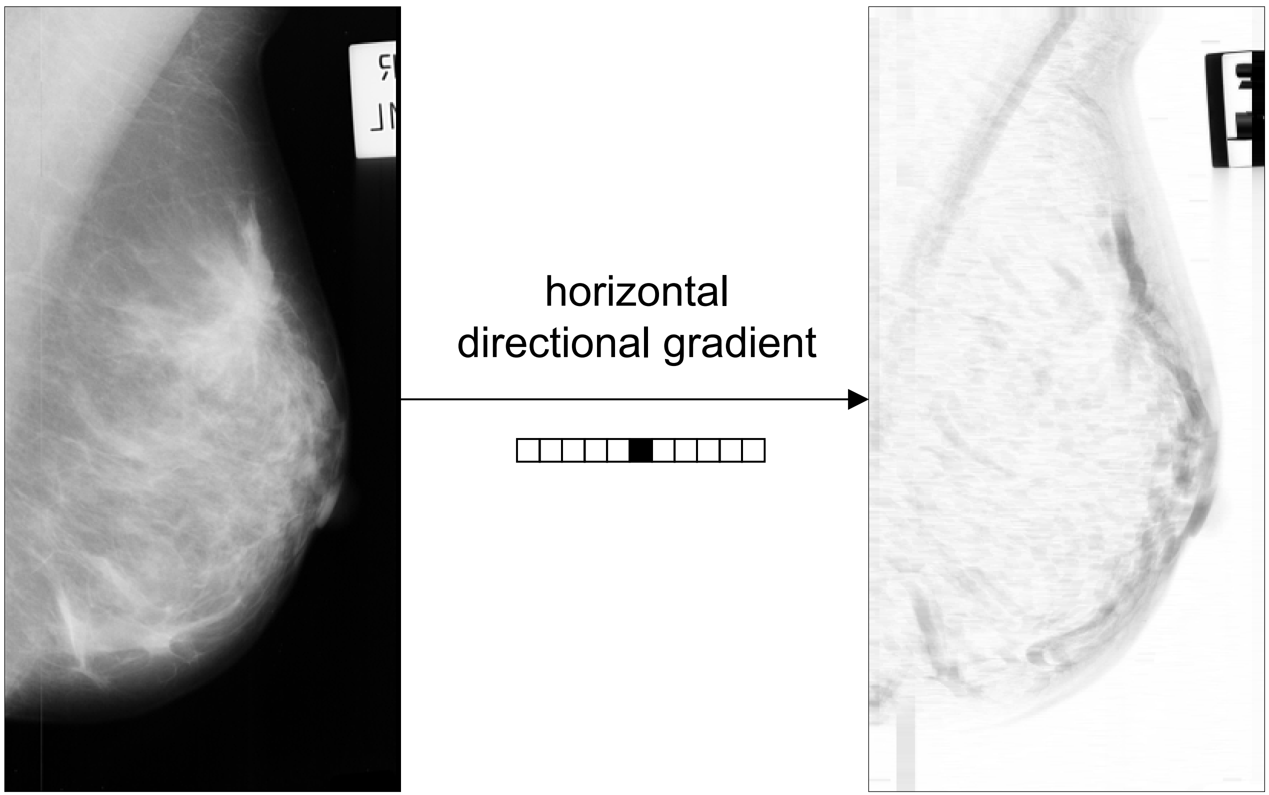
- Directional gradients are defined by replacing the isotropic SE with a line segment *L* in a given direction α :

$$\rho_{L\alpha} = \delta_{L\alpha} - \varepsilon_{L\alpha}$$

- Consider the direction perpendicular to the smallest directional gradient:

  e.g. An image containing a horizontal line would output the highest directional-gradient for all directions, except for the direction of the line → this case would output a null gradient.

Directional Gradients



Multiscale Gradient

- Thick gradients can be used to detect smooth variations between neighboring regions → resulting edges are thick.
  - When the distance separating two boundaries of a region is smaller than the width of the SE the resulting edges merge together.
  - The thickness of the edges obtained from a thick gradient of size *n* can be reduced by an erosion with a SE of size *n-1*

$$\varepsilon_{(n-1)B}(\rho_{nB})$$

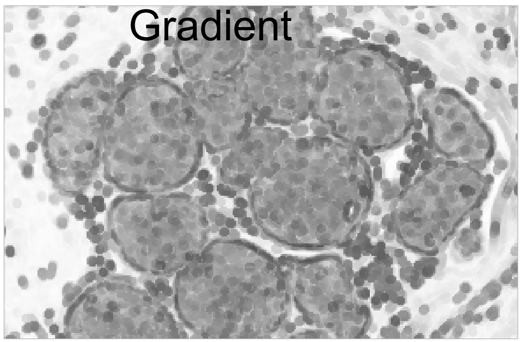
Multiscale Gradient

– When thick gradients from two distinct boundaries merge, the resulting thickness is larger than the width of the SE.
  - These regions can be removed by a WTH of size *n* which is then followed by an erosion of size *n-1*

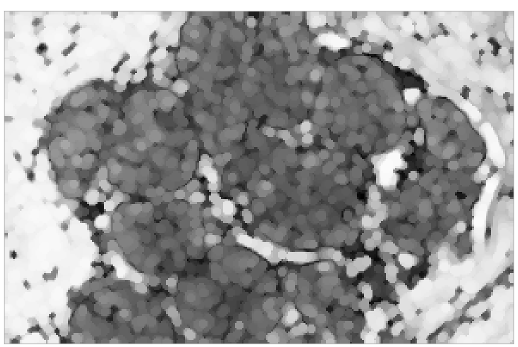
$$\varepsilon_{(n-1)B}(WTH_{nB}(\rho_{nB}))$$

Multiscale Gradient

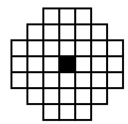
Morphological Gradient



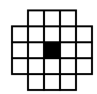
Multiscale Gradient



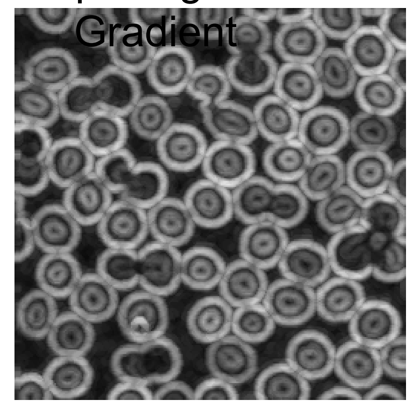
ρ_{nB}



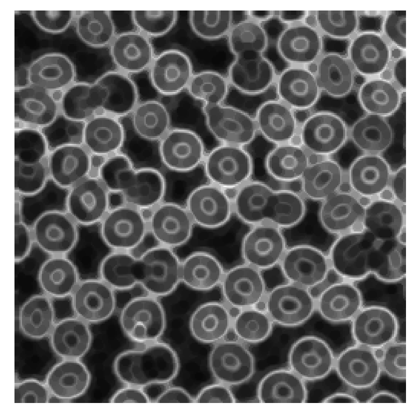
$\varepsilon_{(n-1)B}$



Morphological Gradient



Multiscale Gradient



Morphological Top-Hat

- The choice of a given morphological filter is driven by the available knowledge about the shape, size and orientation of the structures to be filtered.
  - Morphological top-hats proceed *a contrario*.
  - The approach undertaken with top-hats consists of using knowledge about shape characteristics that are not shared by their relevant image structures.
  - It is sometimes easier to remove relevant structures than trying to directly suppress irrelevant objects.

Morphological Top-Hat

- A morphological top-hat is the arithmetic difference between the original image and and its opening, or the closing and the original image.
  - Top-hats enhances image details (thin, sharp positive variations)

White Top-Hat

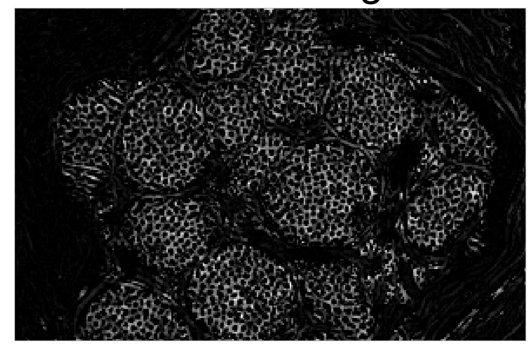
- The white top-hat (WTH) or top-hat by opening of an image f is the difference between the original image f and its opening γ :

$$WTH(f) = f - \gamma(f)$$

  - Peaks in the image are extracted.
  - All structures that cannot contain the SE are extracted by the WTH.

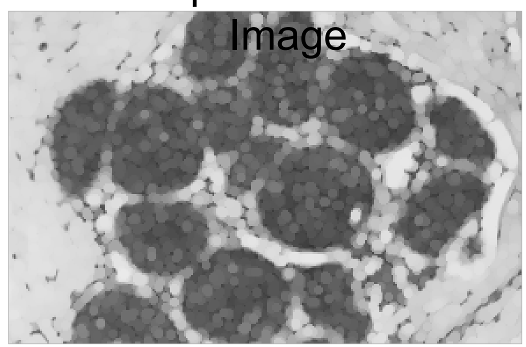
White Top-Hat

WTH Image

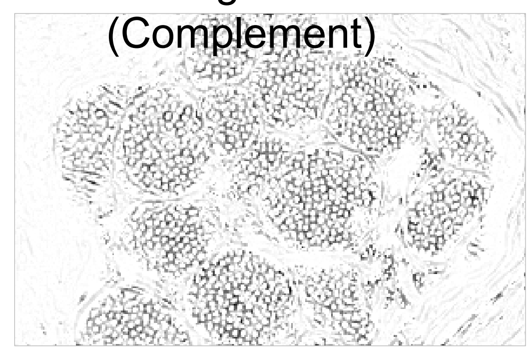


Original Image

Opened Image



WTH Image (Complement)



Black Top-Hat

- The black top-hat (BTH) or top-hat by closing of an image f is the difference between the closing ϕ and the original image f:

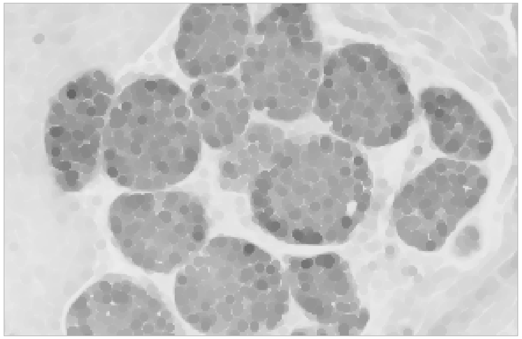
$$BTH(f) = \phi(f) - f$$

  - Troughs in the image are extracted.
  - All structures that cannot contain the SE are extracted by the BTH.

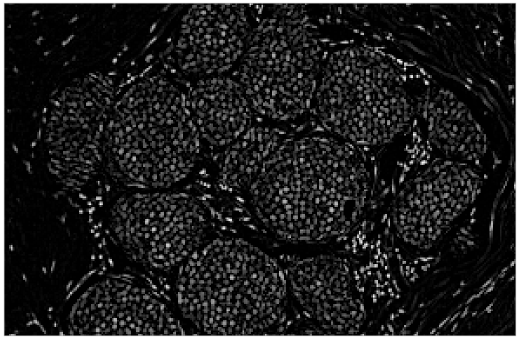
Black Top-Hat

Original Image

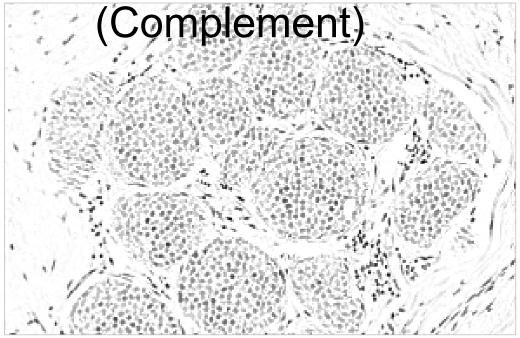
Closed Image



BTH Image



BTH Image (Complement)



Self-Complementary Top-Hat

- The sum of the black and white top-hats extracts all image structures that cannot contain the SE whatever their relative contrast (i.e. peaks and troughs)
- The sum equals the arithmetic difference between the closing and the opening of the image, and is known as the self-complementary top-hat:

$$\varphi = WTH + BTH = \phi - \gamma$$

Top-Hats

- The size and shape of the SE used for the morphoogical top-hat depends on the morphology of the structures to be extracted.

  e.g. to detect bright features of width smaller than *l*, a WTH with a disk SE slightly larger than *l* shoud be considered.

- If the image is corrupted by a high-frequency noise, it must be filtered before using the morphological top-hat:
  - A closing by a small SE before calculating a WTH, and an opening before a BTH

Contrast Enhancement

using Top-Hats

- A simple neighborhood-based morphological contrast operator can be obtained by computing the BTH and WTH of an image in parallel.
- The WTH is then added to the original image (to enhance bright objects), and the BTH is subtracted from the resulting image (to enhance dark objects).

$$\kappa^{TH} = f + WTH_B - BTH_B = f + f - \gamma_B - \phi_B + f$$

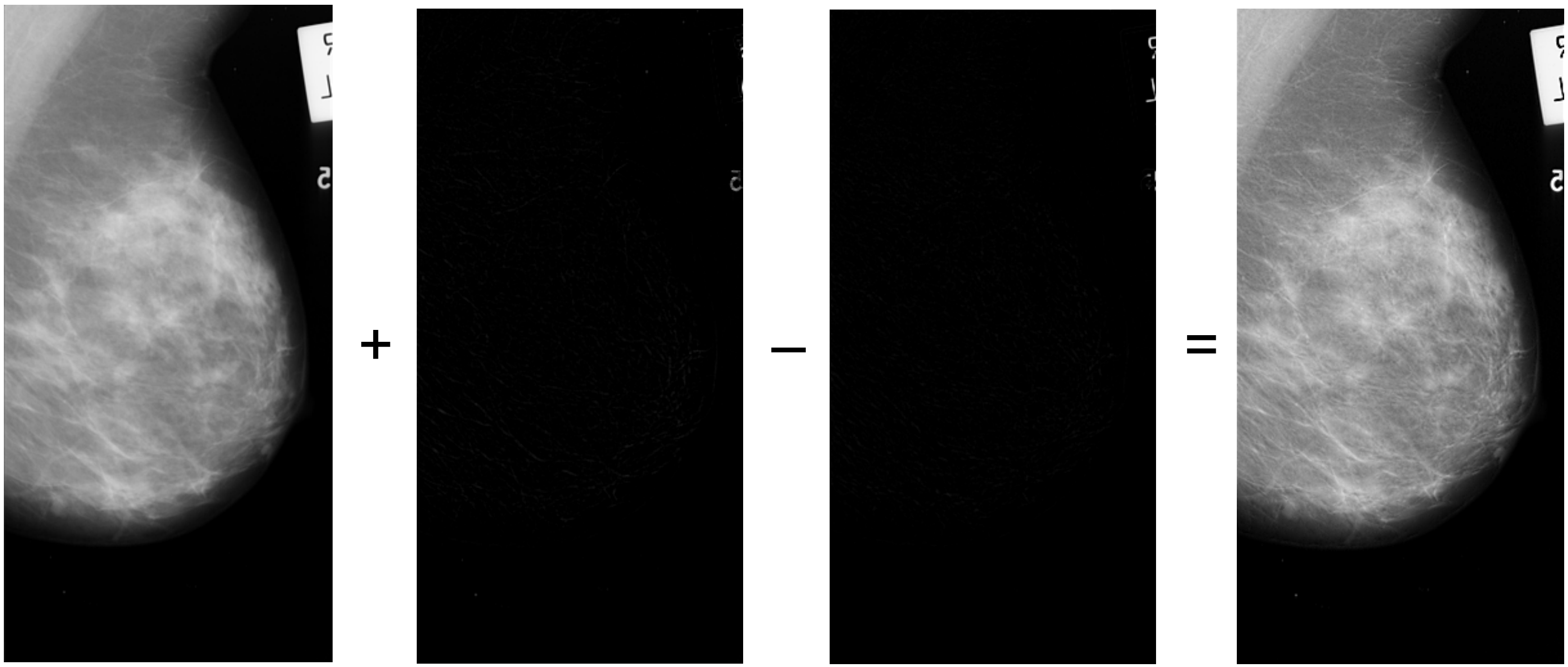
Contrast Enhancement

using Top-Hats

- The output values falling outside the dynamic range of the original image $[t_{min}, t_{max}]$ are set to t_{min} or t_{max} depending on whether they fall below or above the dynamic range.

Contrast Enhancement

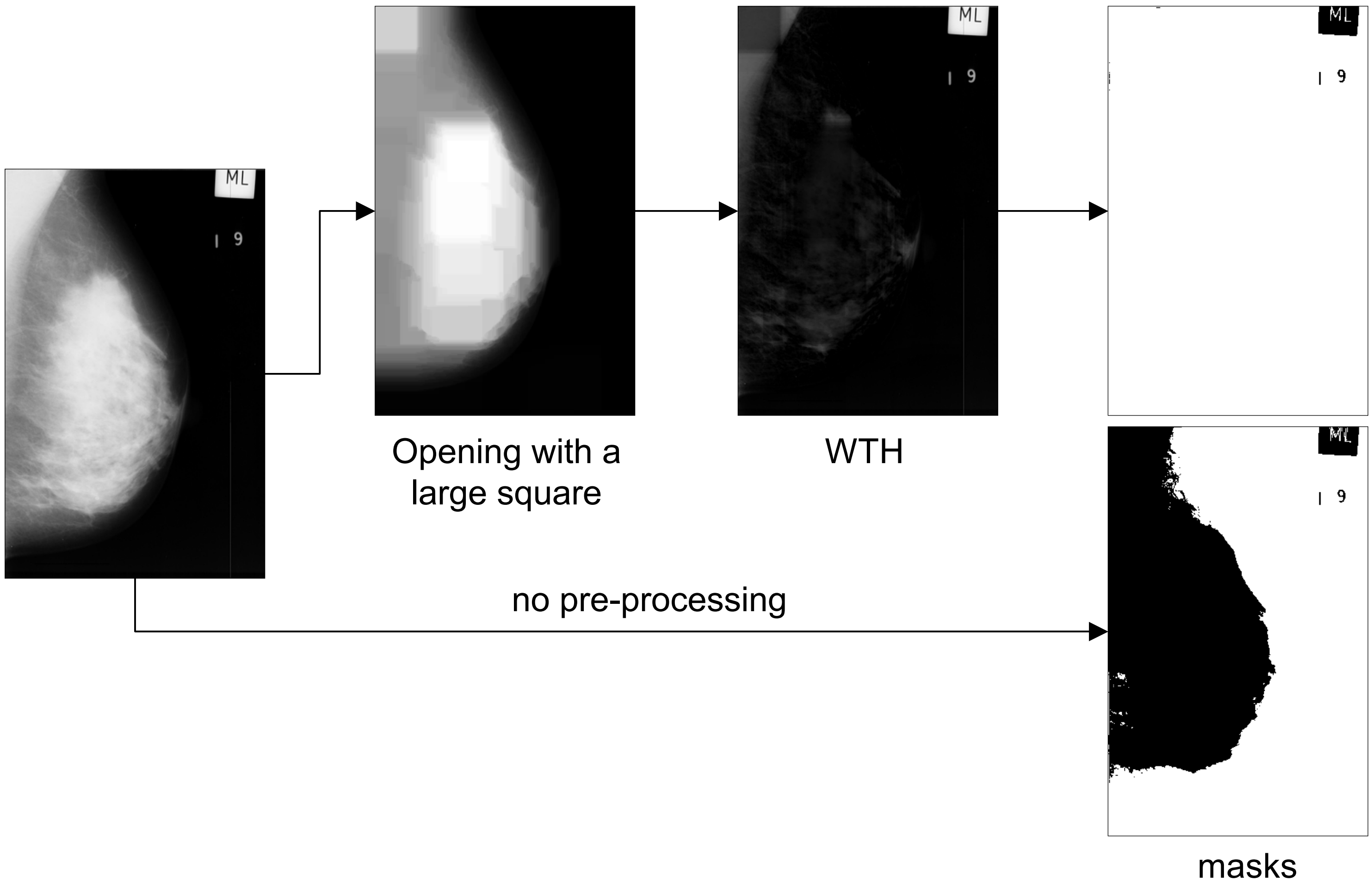
using Top-Hats



f WTH BTH

Difference Masks

using Top-Hats



Alternating Sequential Filter

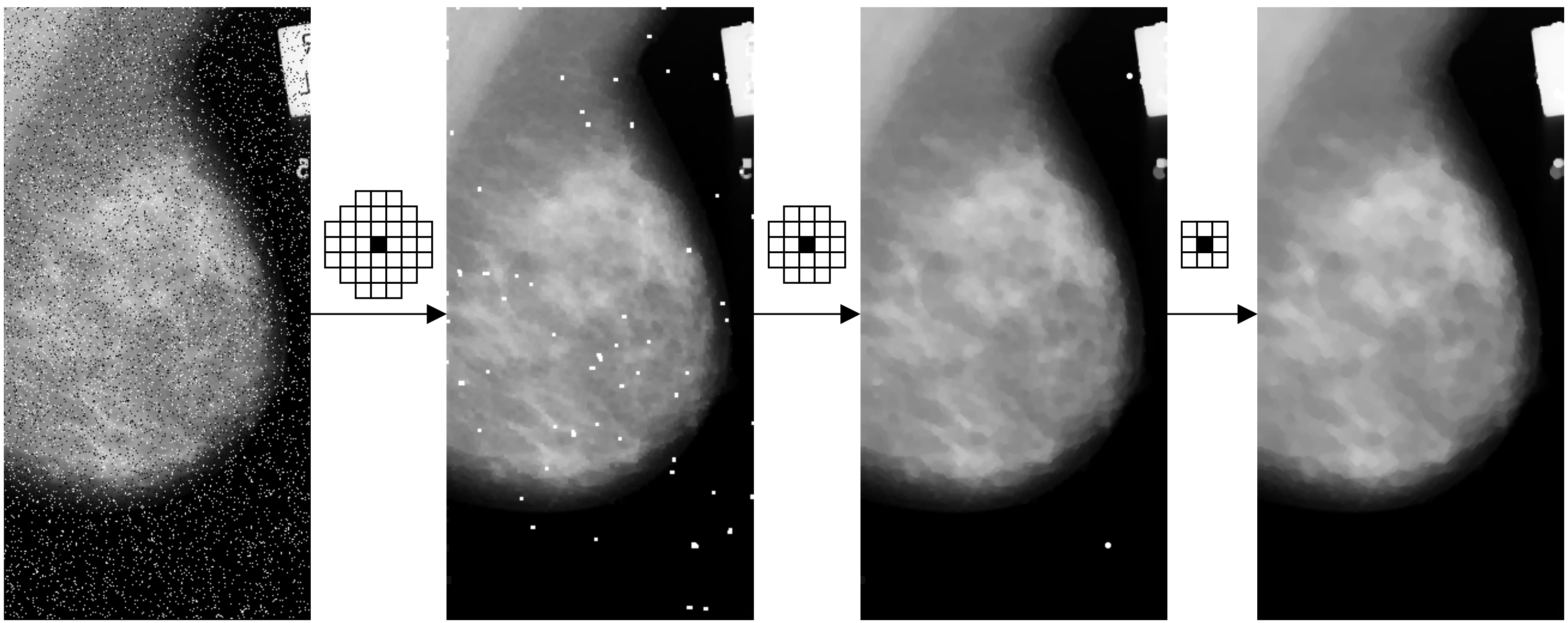
- Filtering of an image containing dark and bright noise can be achieved by a sequence of either close-open or open-close filters.
  - When the level of noise is high (it contains noisy structures over a wide range of scales)
  - A unique close-open or open-close filter with a large SE does not lead to acceptable results.
  - A solution to this problem is to alternate closings and openings, beginning with a small SE and then proceeding with ever-increasing SE until a given size is reached.

Alternating Sequential Filter

- This sequential application of open-cose (or close-open) filters is called an Alternating Sequential Filter.

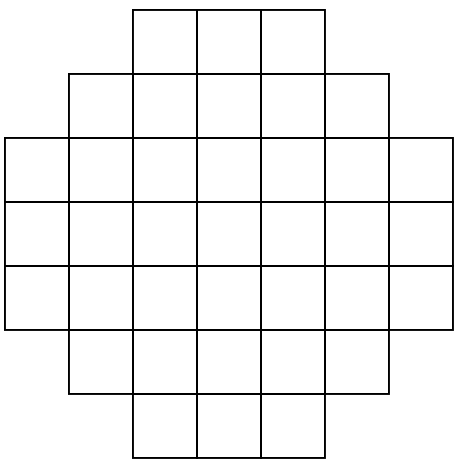
Alternating Sequential Filter

A Sequence of Open-Close Filters

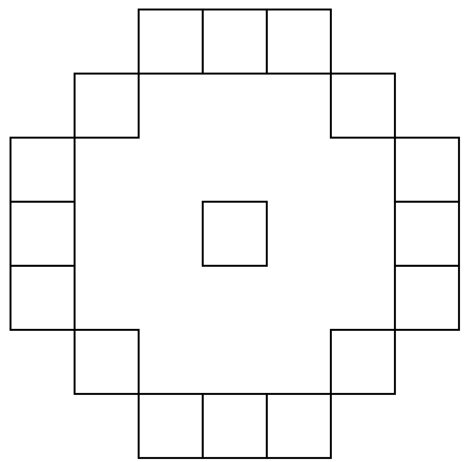


SE Shapes

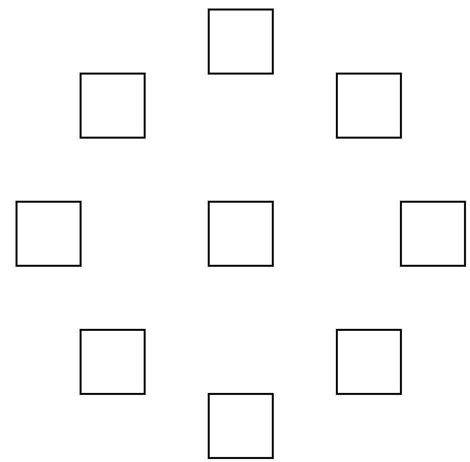
Round Disk



“Empty Disk”

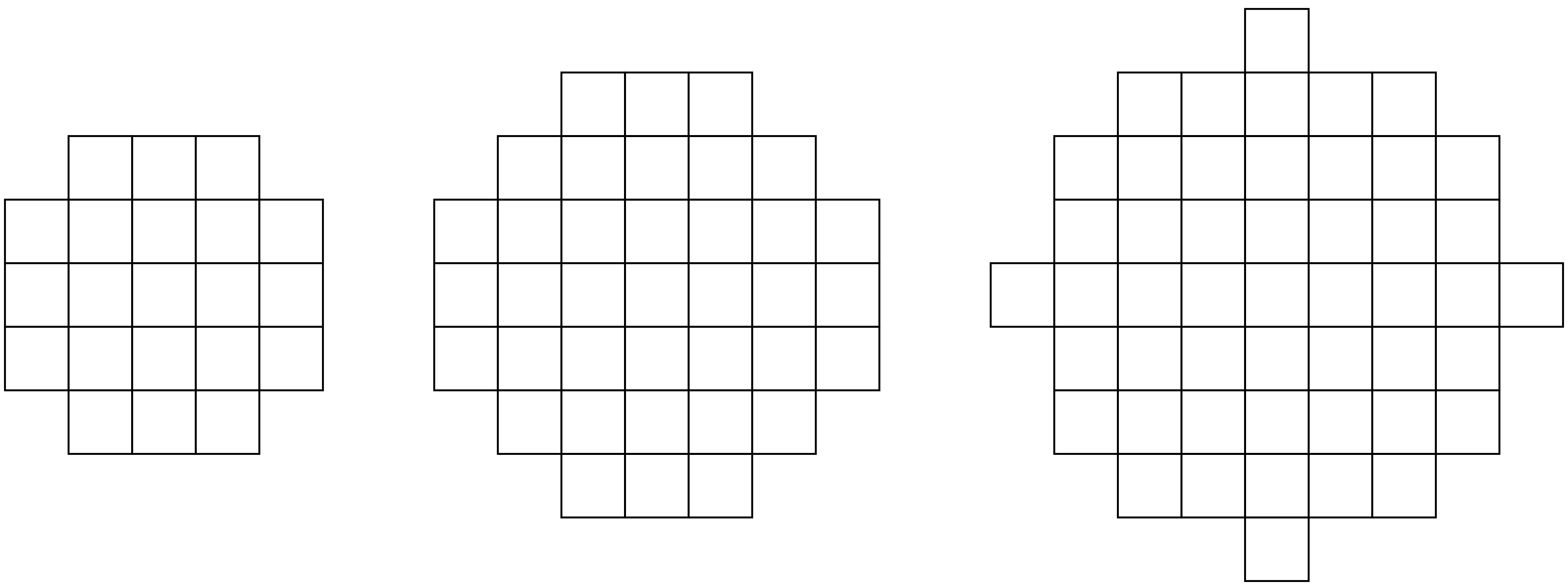


“Sparse” Disk



SE Shapes

Differing sized "disks"



Other Morphological Operations

- Morphological Sharpening
- Geodesic Transformations
  - e.g. morphological reconstruction
- Morphological Filtering
- Morphological Segmentation
  - e.g. Watershed segmentation
- Morphological Classification