Morphological Image Analysis

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What is Morphological Analysis?

- The term "morphological analysis" describes a range of non-linear image processing techniques that deal with the shape or morphology of features in an image.
  - The word morphology refers to form and structure.
  - Known as "mathematical morphology"
  - Most morphological analysis techniques operate on binary images

Morphological Analysis

- Uses for morphological analysis include:
  - Noise reduction and feature detection.
    - The objective is that noise be reduced as much as possible without eliminating essential features.
  - Analysis of connectivity of components
  - Object selection using geometric features

Structuring Elements

- Morphological operations use a small shape or template known as a structuring element (SE).
  - The structuring element is positioned at all possible locations in the image and is compared to the corresponding neighbourhood of pixels.
  - Morphological operations differ in how they carry out this comparison.
    - Some test whether the SE “fits” within the neighborhood, others test whether it “hits” or intersects the neighborhood.

Structuring Elements

- The structuring element applied to a binary image can be represented as a small matrix of pixels, each with a value of 1 or 0.
  - The dimensions of the matrix determine the size of the SE, and its shape is determined by the pattern of ones and zeros.

$$SE = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Structuring Elements

- A structuring element has an origin.
  - This should be indicated when representig the structuring element

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & \mathbf{1} & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

  - The origin allows the positioning of the SE at a given point or pixel: a SE at point *x* means that its origin coincides with *x.*

Structuring Elements

- As with convolution kernels, it is common for structuring elements to have odd dimensions
- A SE is analogous to the kernel in convolution
- The shape and size of the SE must be adapted to the geometric properties of the image objects of interest:
  - e.g. linear SEs are suited to the extraction of linear objects.

Structuring Elements

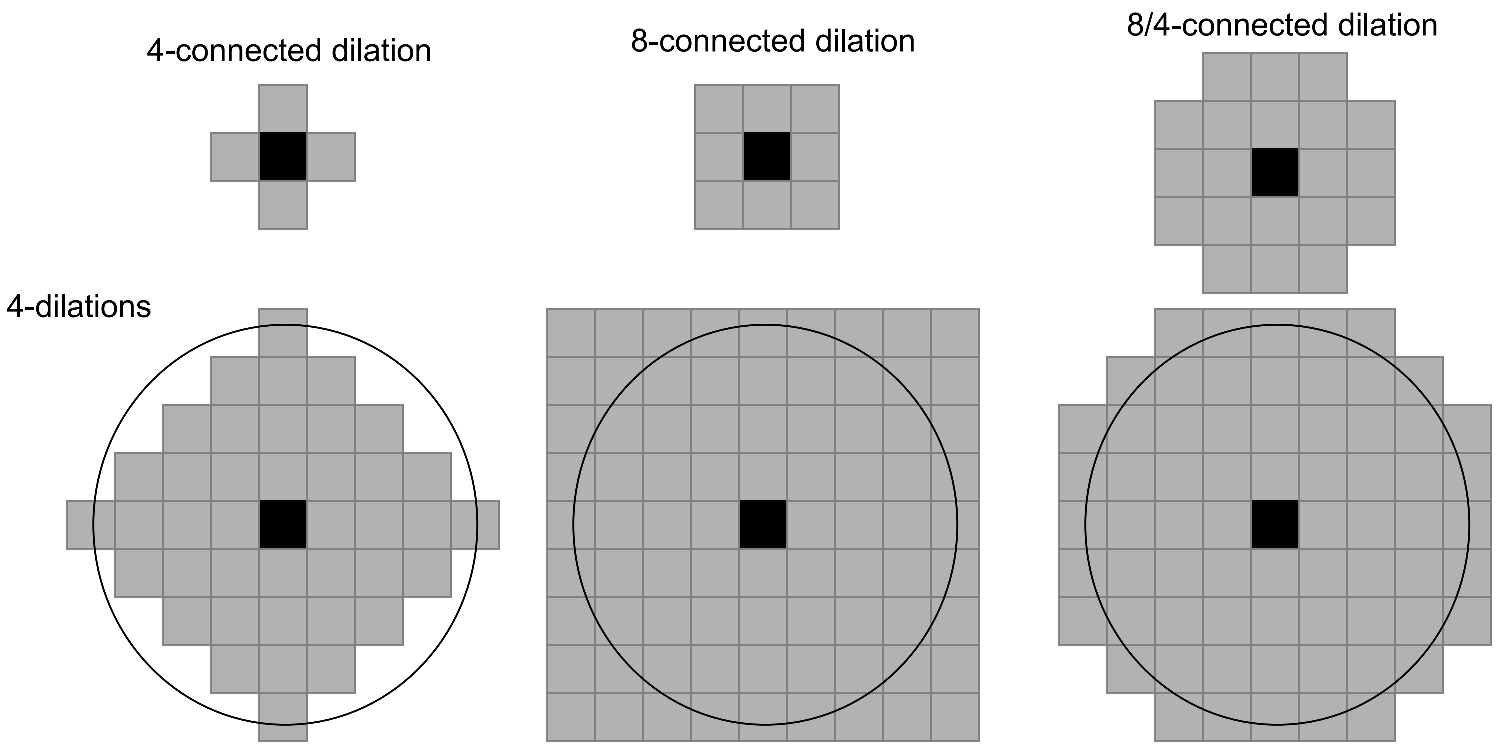
- The shape of the SE is usually chosen according to some priori knowledge about the geometry of the relevant image structures.
  - A SE takes into account a number of factors: shape, size, and orientation
  - For example, line segments are often used to remove or extract elongated structures.
    - Two parameters: length and orientation

Structuring Elements

- SEs are used to investigate the morphology of image objects.
  - It is recommended to use *n*-dimensional SEs
  - Often referred to as **flat** SEs because they have two dimensions in the case of 2D images.
  - The shape of the SEs does not depend on the scaling of the image gray levels.
  - *n*+1 dimensional SEs are called non-flat or grayscale

Disks

- Isotropic disks can only be approximated
  - The larger the neighborhood size, the larger the number of possible approximations.



Adaptive SE

- An adaptive SE involves adapting the shape of the SE from one pixel to another.
- The total number of pixels in the SE ranges from a lower to an upper limit within a given neighborhood.
- The SEs are chosen so as to minimize a statistical criterion.

Composite SE's

- A composite or two-phase SE contains two non-overlapping SEs sharing the same origin.

  e.g. Hit-or-miss transformations: the first SE is used for erosion, the second for dilation

Fitting and Hitting

- When a SE is placed in a binary image, each of the pixels in the SE is associated with the corresponding pixel of the neighborhood under the structuring element.
- Similar to the process of convolution, however the operation is logical rather than arithmetic.
- Fitting:
  - The SE is said to **fit** an image if, for each of its pixels that is set to 1, the corresponding image pixel is also 1.

Fitting and Hitting

- Hitting:
  - The SE is said to **hit**, or intersect an image if, for any of its pixels that is set to 1, the corresponding image pixel is also 1.

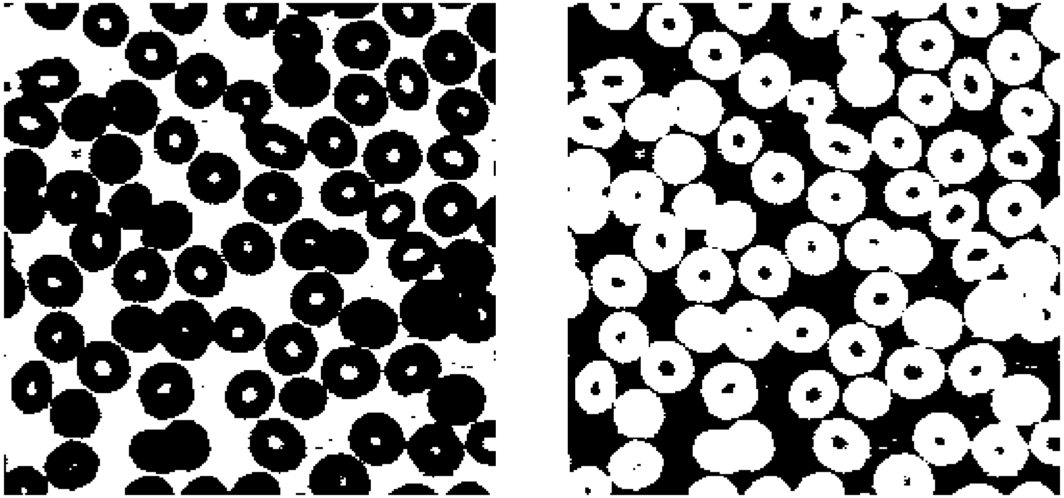
Elementary Operations

- The most primitive morphological operations are erosion and dilation.
  - Both operations can be applied iteratively to erode or dilate by layers.

Complement (NOT)

- The complement of a binary image:

$$A^C = \{\alpha \mid \alpha \notin A\}$$



Intersection (AND)

- The intersection of two binary images **A** and **B** is the set of non-zero pixels common to both images:

$$A \bigcap B = \left\{ \alpha \mid \alpha \in A \wedge \alpha \in B \right\}$$

Union (OR)

- The union of two binary images **A** and **B** is the set of non-zero pixels in either images:

$$A \bigcup B = \left\{ \alpha \mid \alpha \in A \vee \alpha \in B \right\}$$

Translation

- The translation of A_p of a set of pixels A by a position vector p is defined by:

$$A_x = \{\alpha + x \mid \alpha \in A\}$$

Minkowski Operations

- Minkowski Addition

$$A \oplus B = \bigcup_{\beta \in B} (A + \beta)$$

- Minkowski Subtraction

$$A \ominus B = \bigcap_{\beta \in B} (A + \beta)$$

Erosion and Dilation

- Erosion and dilation are dual morphological operations which cause a reduction or enlargement in the size of regions respectively.

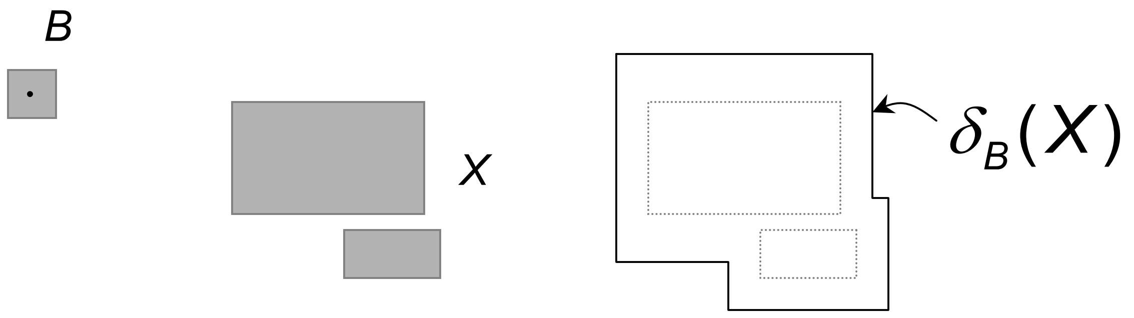
Morphological Dilation

- Morphological dilation expands or dilates an image:
  - It shrinks the holes enclosed by a single region and make the gaps between different regions smaller.
  - It tends to fill in any small intrusions into a regions boundaries.

Dilation

- The dilation of a set **X** by a structuring element *B* is denoted by $\delta_B(X)$ and is defined as:

$$\delta_B(X) = \{x \mid B_x \cap X \neq 0\}$$

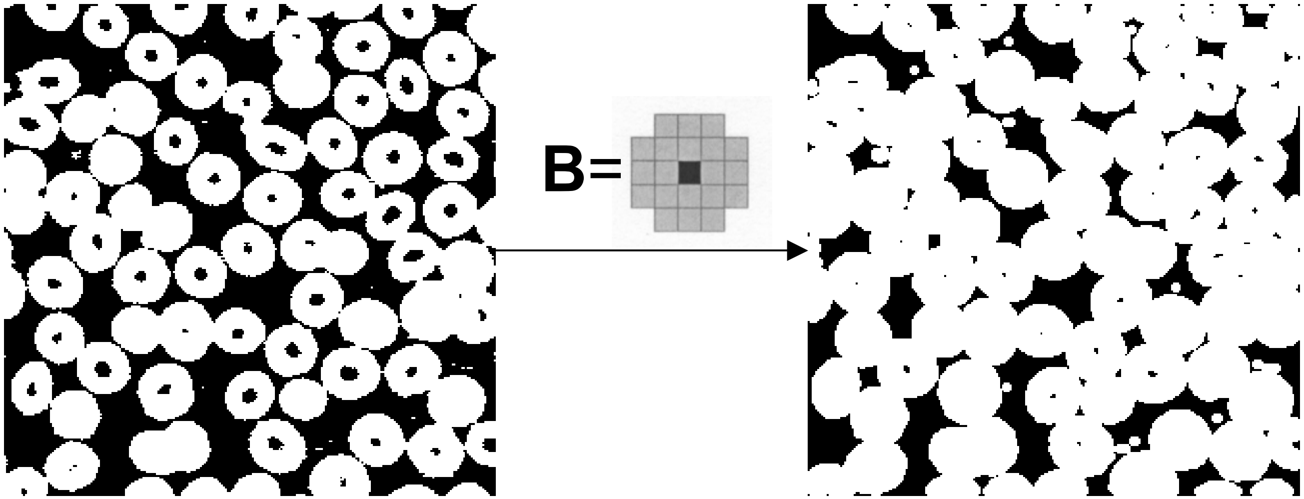


Dilation

- The union of the translations of an image **A** by the pixels of the SE **B** is called the dilation of **A** by **B** and is given by:

$$\delta_B(A) = A \oplus B = \bigcup_{\beta \in B} A_\beta$$

Dilation



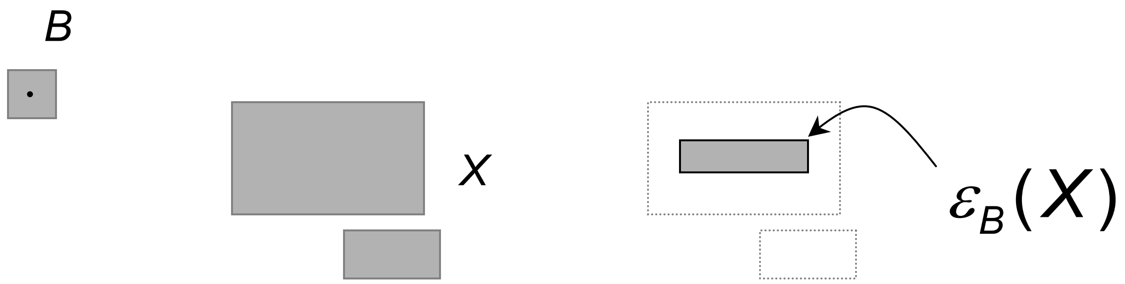
Morphological Erosion

- The opposite of dilation is erosion.
- Morphological erosion shrinks or erodes an image:
  - It expands the holes enclosed by a single region and make the gaps between different regions larger.

Erosion

- The erosion of a set **X** by a structuring element B is denoted by $\varepsilon_B(X)$ and is defined as:

$$\varepsilon_B(X) = \{x \mid B_x \subseteq X\}$$



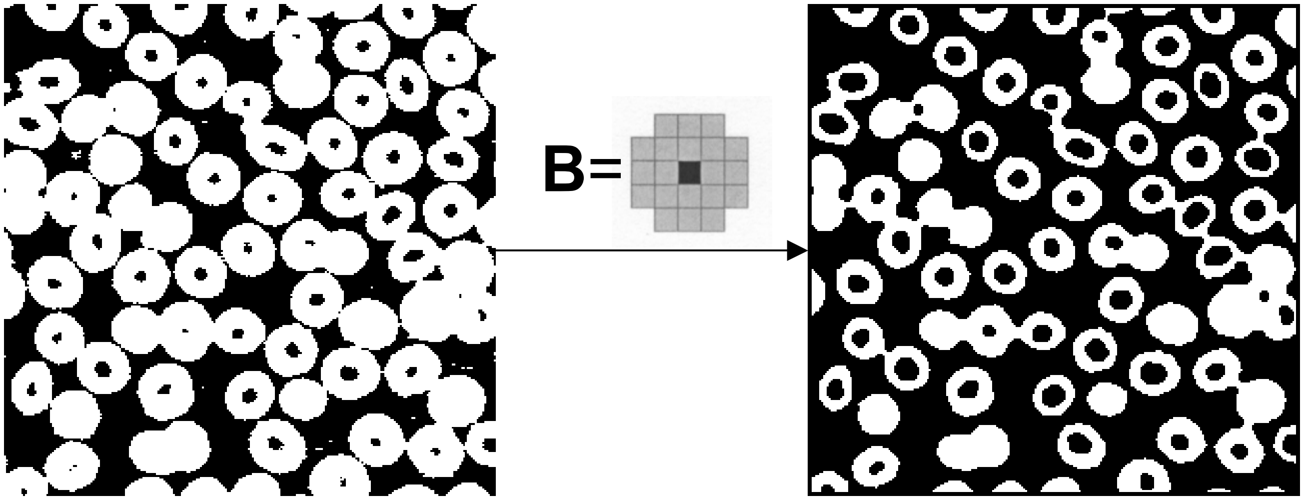
Erosion

- The intersection of the translations of an image **A** by the pixels of a SE **B** is called the erosion of **A** by **B** and is given by:

$$A \ominus B = \left\{ p \mid B_p \subseteq A \right\}$$

$$\varepsilon_B(A) = A \ominus B = \bigcap_{\beta \in B} (A + \beta)$$

Erosion



Compound Operations

- Compound morphological operations are combinations of the elementary operations of erosion and dilation.
  - morphological closing
  - morphological opening

Morphological Opening

- Morphological opening involves the application of erosion, followed by dilation.
  - The effect is to smooth boundaries, to break narrow isthmuses, and to eliminate small noise regions.
  - Separate connected objects, remove small objects

Opening

- Morphological opening of an image **A** by the pixels of a SE **B** is is given by:

$$A \circ B$$

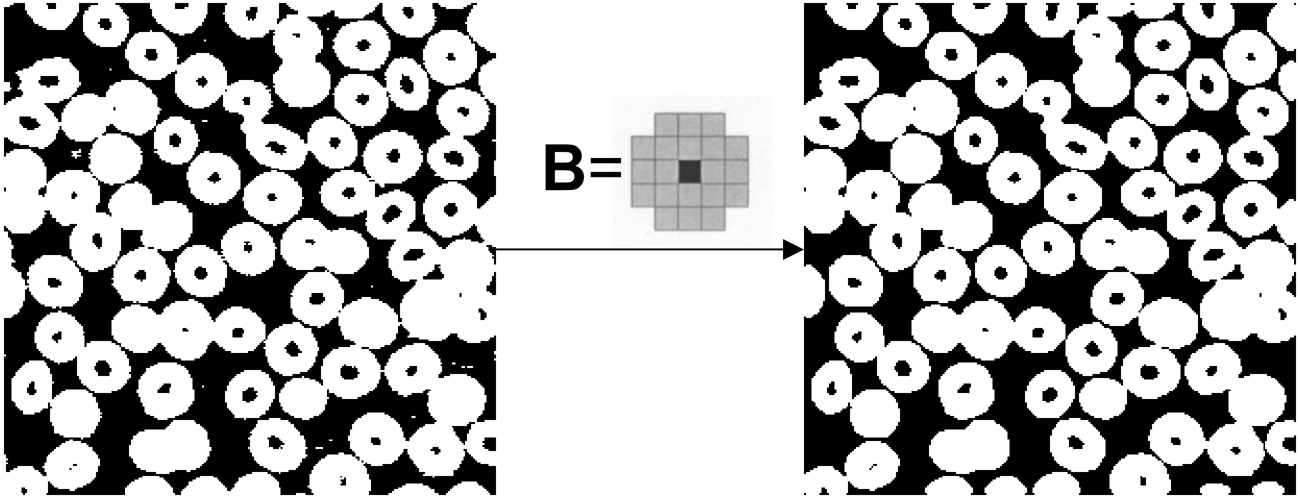
and is defined as an erosion, followed by a dilation:

$$\gamma_B(A) = A \circ B = (A \ominus B) \oplus B$$

Opening

- Opening is said to be idempotent.
  - This means that, once an image has be opened, subsequent openings with the same SE have no further effect on that image.

Opening



Morphological Closing

- Morphological closing involves the application of dilation, followed by erosion.
  - The effect is to smooth boundaries, to join narrow breaks, and to fill small holes caused by noise.
  - Connect disconnected objects

Closing

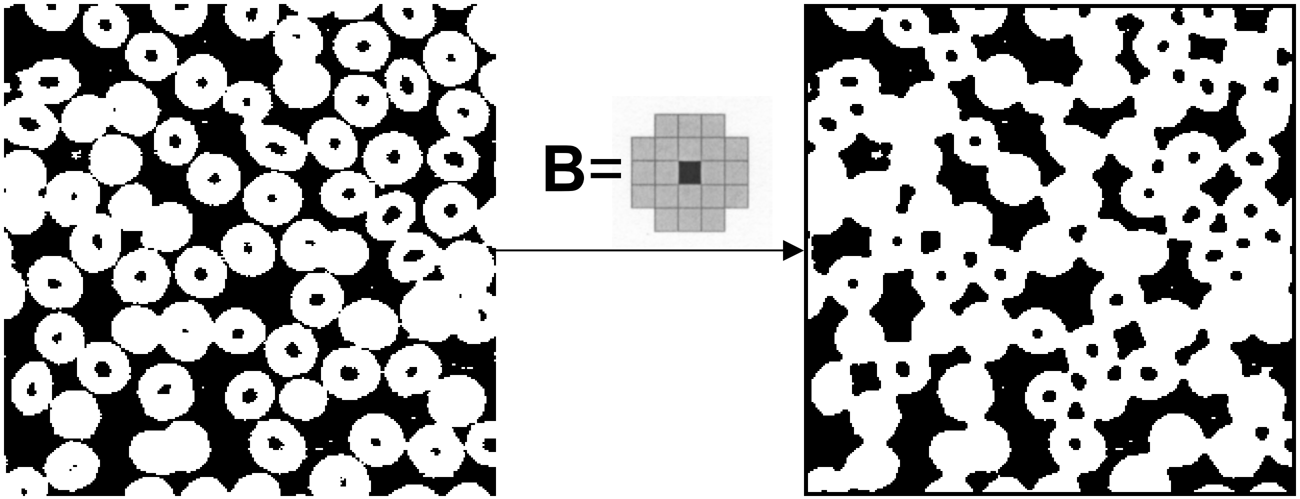
- Morphological closing of an image **A** by the pixels of a SE **B** is is given by:

$$A \bullet B$$

and is defined as an erosion, followed by a dilation:

$$\phi_B(A) = A \bullet B = (A \oplus B) \ominus B$$

Closing



Opening versus Closing

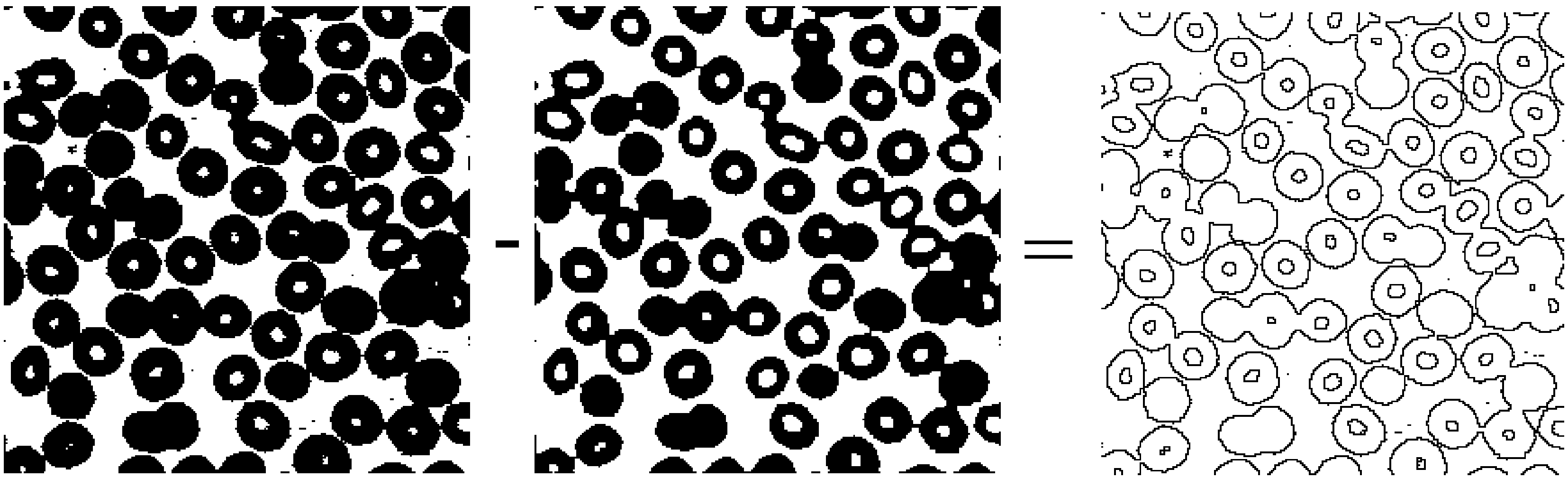
- The difference between opening and closing is in the initial iteration, erosion, or dilation.
- The choice of operation depends on the image and the objective.
  - Opening is used when the image has many small regions. It is not used for narrow regions where there is a chance that the initial erosion operation might disconnect regions.
  - Closing is used when a region has become disconnected and the desire is to restore connectivity. It is not used when different regions are located closely such that the first iteration of dilation might connect them.

Contour Extraction

- One application of erosion is contour extraction. An image eroded using cellular erosion contains regions lacking boundary pixels.
  - The contours are extracted by subtracting the eroded image from the original image.

$$G = A - (A \ominus B)$$

Contour Extraction



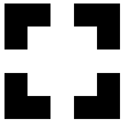
“Hit or Miss” Transform

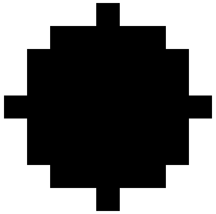
- The hit-or-miss transformation serves to detect features in the image that match the shape of the structuring element.
  - This operation requires a matched pair of SEs $\{\mathbf{b_1},\mathbf{b_2}\}$ that probe the inside and outside of objects in the image respectively.

$$\mathbf{A} \otimes \{\mathbf{b_1},\mathbf{b_2}\} = (\mathbf{A} \ominus \mathbf{b_1}) \cap (\mathbf{A^c} \ominus \mathbf{b_2})$$

"Hit or Miss" Transform

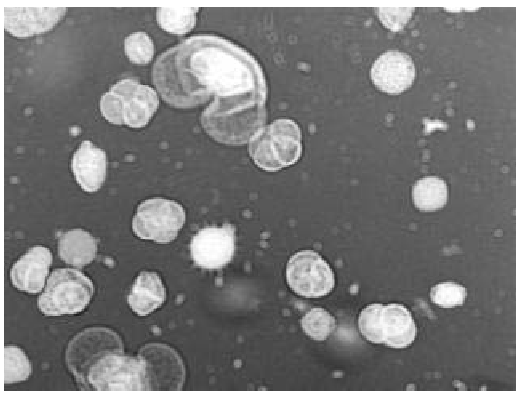
- For example:
  - Identify blobs with a radius of at least 2, but less than 4 in the pollen image. These regions totally enclose a disc of radius 2, contained in the 5 x 5 kernel named "hit", and in turn, fit within a hole of radius 4, contained in the 9 x 9 array named "miss".



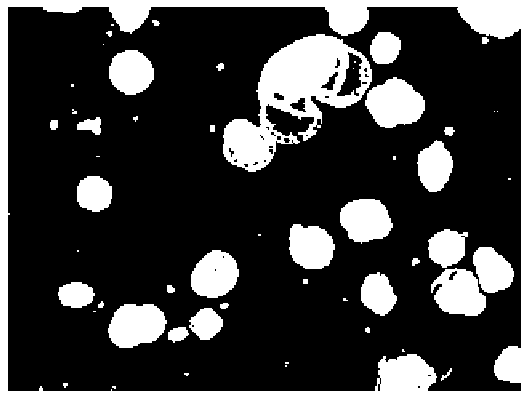


“Hit or Miss” Transform

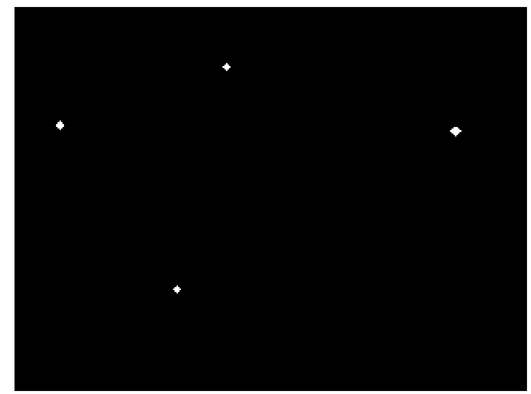
Original Image



Binary Image



Objects with $2 \leq r<4$



- Disadvantages of Hit-or-Miss
  - sensitive to noise, requires exact shape.

Conditional Dilation

- Conditional dilation is used in certain occasions when it is desirable to dilate an object in such a way that certain pixels remain immune.
  - Propagation is a form of conditional dilation which grows a seed **S** into a mask **A**

$$S^{(k)} = \left[S^{(k-1)} \oplus B \right] \cap A \quad \text{until} \quad S^{(k)} = S^{(k-1)}$$

  - Applications include finding specific objects and closing holes.

Skeletonization

- Morphological skeletonization reduces a region to its minimum number of connected "1"-valued pixels
  - Sometimes known as thinning.
  - A form of conditional erosion, which is an erosion under one or more of the following conditions:
    - Do not remove a single pixel
    - Do not break the connectivity
    - Do not remove an end-pixel

Skeletonization

- Skeletonization reduces a region shape to lines that are symmetrically contained within the shape.
  - These "skeletons" approximate centre lines with respect to original region boundaries.
  - Although the thinning can be applied to binary images containing regions of any shape, it is most suited for elongated, as opposed to "blob"-like shapes.

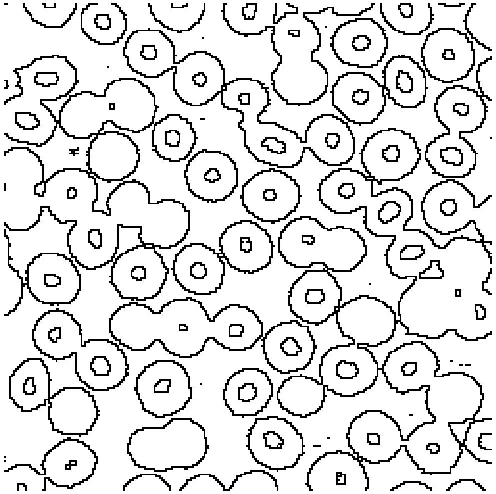
    e.g. chromosomes are classified as X and Y on the basis of their characteristic elongated shapes and patterns of line branching.

Skeletonization

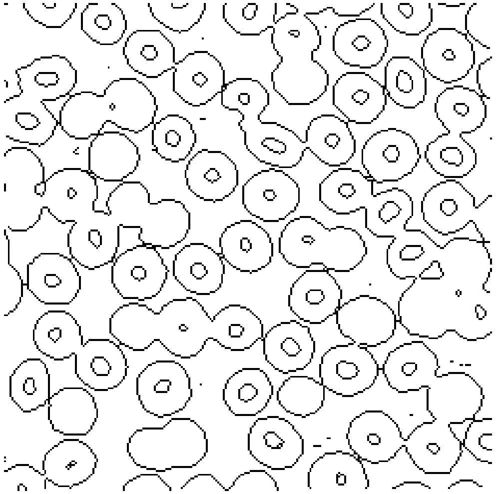
- Objectives of skeletonization :
  - Connected image regions must thin to connected line structures. The thinning operation should preserve the connectivity of the original.
  - The thinned result should be minimally eight connected.
  - Approximate end-line locations should be maintained.
  - The thinning result should approximate medial lines.
  - Extraneous “spurs” introduced by thinning should be minimised.

Skeletonization

Extracted Boundaries



Skeletonization



Skeletonization

- Morphological pruning reduces short branches extruding from a region after skeletonization.

Cellular Processing

- In morphological processing, new pixel values are assigned on the basis of boolean operations applied to a designated neighborhood of each pixel.
- Cellular processing relies on arithmetic evaluations of pixel values over a designated neighborhood and subsequent comparison with preselected thresholds.

Cellular Processing

- For each pixel, the result depends on two calculations:
  - ❶ The number of neighborhood 1-valued pixels is calculated:

    $$\phi(i,j,n) = \sum_{(n,m)\in\eta} I(n,m)$$

    where η describes the neighborhood pixels around the centre pixel.

Cellular Processing

❷ The number of eight-connected groups of pixels in the neighborhood

$$\chi(i,j,n) = \sum_{(n,m)\in\eta} I(n,m)$$

- The results are compared with predefined thresholds T_ϕ and T_χ, and the centre pixel is set depending on the outcome.

Cellular Erosion

- Cellular erosion is used to remove 1-valued pixels at boundaries of regions and to increase the size of holes.
- Erosion entails setting a 1-valued pixel to 0 if the number of neighborhood 1-valued pixels falls short of the preselected threshold and if the following connectivity criterion is met:

$$\alpha_C(i_0, j_0) = \begin{cases} 0 & \text{if } I(i_0, j_0) = 1 \text{ and } \phi(i, j, \eta) < T_\phi \text{ and } \chi(i, j, \eta)\, \mathbf{op}\, T_\chi \\ 1 & \text{otherwise} \end{cases}$$

Cellular Erosion

where **op** stands for an operand (>,≤,=)

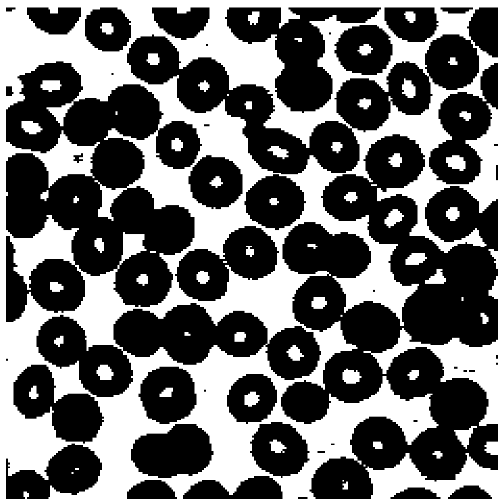
- The most frequent choice of a threshold T_ϕ for erosion is half of the total number of neighborhood pixels.
  - This tends to remove rough spurs and corners from region boundaries.
  - A higher value of T_ϕ leads to faster and coarser erosion.
  - A lower value of T_ϕ leads to a slower and finer erosion.

Cellular Erosion

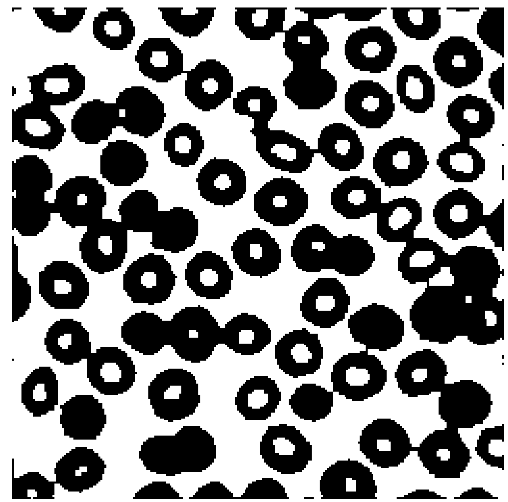
- In choosing the operand **op** and the threshold T_χ for connectivity, determine whether to tolerate a change in connectivity.
  - If connectivity is of no concern, $\chi(i,j,n) \leq T_\chi \equiv 8$
  - If connectivity does matter:
    - If the centre pixel is to be set to 0 only if connectivity remains unchanged, the connectivity condition should be $\chi(i,j,n) = 1$
    - If the centre pixel is set to zero only if connectivity does change, the criterion should be $\chi(i,j,n) > 1$

Cellular Erosion

Original Image



Cellular Erosion



Cellular Dilation

- Cellular dilation is used to add 1-valued pixels at region boundaries or to fill holes.
- Dilation entails setting a 0-valued pixel to 1 if the number of neighborhood 1-valued pixels equals or exceeds the preselected threshold and if the following connectivity criterion is met:

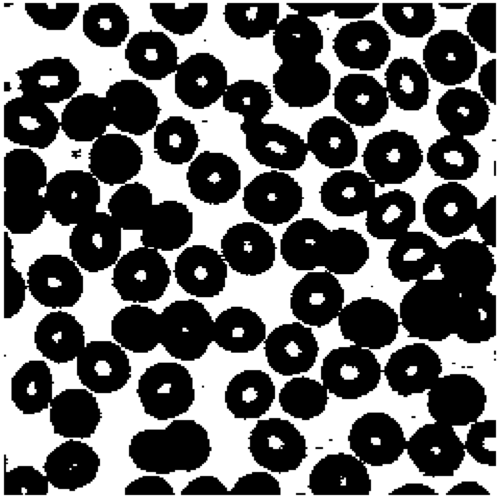
$$\delta_D(i_0, j_0) = \begin{cases} 0 & \text{if } I(i_0, j_0) = 0 \text{ and } \phi(i, j, \eta) \geq T_\phi \text{ and } \chi(i, j, \eta) \,\mathbf{op}\, T_\chi \\ 1 & \text{otherwise} \end{cases}$$

Cellular Dilation

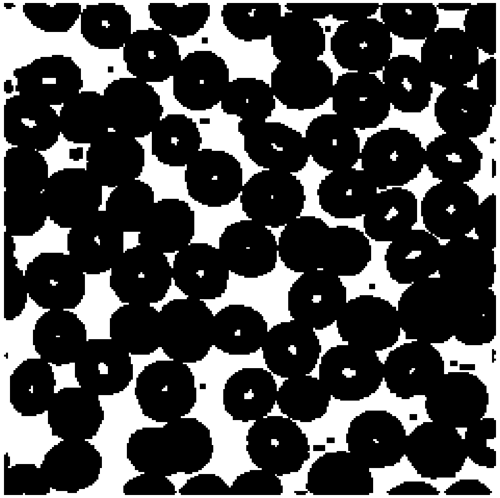
- The most frequent choice of a threshold T_ϕ for erosion is half of the total number of neighborhood pixels.
  - This choice ensures that indentations at the region boundaries are to be filled in.
  - A lower value of T_ϕ leads to faster and coarser erosion.
  - A higher value of T_ϕ, up to eight, leads to a slower and finer erosion; a value of eight yields no dilation.

Cellular Dilation

Original Image



Cellular Dilation



Cellular Opening and Closing

- Cellular processing usually involves several iterations of erosion and dilation operations.

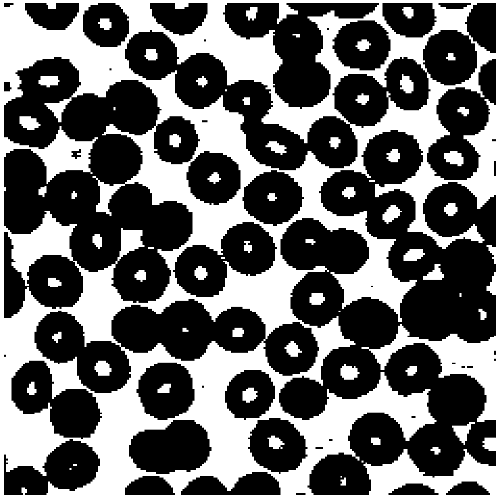
  e.g. to perform smoothing, erosion and dilation iterations are alternated. The larger the number of iterations, the more pronounced the smoothing.

  – Cellular Opening: erosion preceding dilation will tend to eliminate regions of 1-valued noise, but will also eliminate long or thin features.

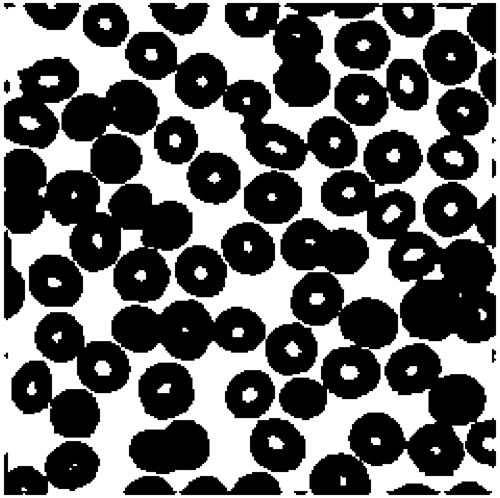
  – Cellular Closing: dilation precedes erosion, this will eliminate 0-valued holes, but will tend to smooth sharp features.

Cellular Opening and Closing

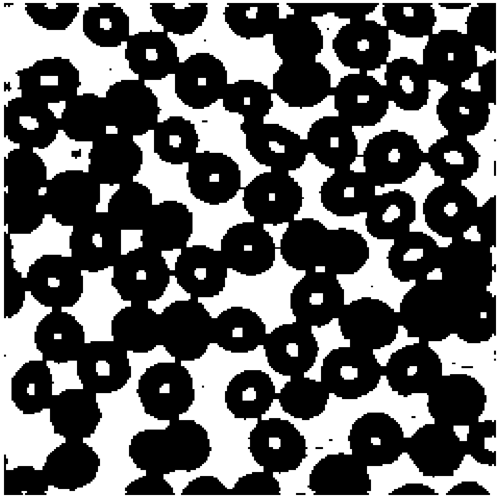
Thresholded Image



Cellular Opening



Cellular Closing



Coefficient and Depth Parameters

- Coefficients:
  - One effect of different coefficient values for T_ϕ is to alter the rate at which features grow or shrink, and to some extent control the isotropy of the result.
    - Classical erosion would use a value of $T_\phi = 0$
    - For values of T_ϕ from 5 to 7, only isolated noise pixels would be removed.
    - Alternating between two coefficients, e.g. 0 and 1

Coefficient and Depth Parameters

- Depth:
  - The depth is the number of iterations of the operation.
  - The greater the depth, the greater the effect.

References

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